Performance evaluation in $dtsPBC$

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Abstract: In [Tar06], we constructed a discrete time stochastic extension $dtsPBC$ of finite Petri box calculus PBC [BDH92] enriched with iteration.

The step operational semantics was defined in terms of labeled probabilistic transition systems.

The denotational semantics was defined in terms of a subclass of labeled DTSPNs (LDTSPNs) called discrete time stochastic Petri boxes (dts-boxes).

In this talk, we propose the method of modeling and performance evaluation based on stationary behaviour analysis applied to the shared memory system.

Keywords: stochastic process algebra, Petri box calculus, iteration, discrete time, stationary behaviour, performance evaluation.

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Introduction

Algebra PBC and its extensions

- *Petri box calculus PBC* [BDH92]
- *Time Petri box calculus $tPBC$* [Kou00]
- *Timed Petri box calculus $TPBC$* [MF00]
- *Stochastic Petri box calculus $sPBC$* [MVCC03]
- *Ambient Petri box calculus $APBC$* [FM03]
- *Arc time Petri box calculus $atPBC$* [Nia05]
- *Generalized stochastic Petri box calculus $gsPBC$* [MVCR08]
- *Discrete time stochastic Petri box calculus $dtsPBC$* [Tar06]

Syntax

The *set of all finite multisets* over X is $\mathbb{N}_f^X$.

$Act = \{a, b, \ldots\}$ is the set of *elementary actions*.

$\widehat{Act} = \{\hat{a}, \hat{b}, \ldots\}$ is the set of *conjugated actions (conjugates)* s.t. $a \neq \hat{a}$ and $\hat{\hat{a}} = a$.

$\mathcal{A} = Act \cup \widehat{Act}$ is the set of *all actions*.

$\mathcal{L} = \mathbb{N}_f^{\mathcal{A}}$ is the set of *all multiactions*.

The *alphabet* of $\alpha \in \mathcal{L}$ is defined as $\mathcal{A}(\alpha) = \{x \in \mathcal{A} \mid \alpha(x) > 0\}$.

An *activity (stochastic multiaction)* is a pair (α, ρ), where
$\alpha \in \mathcal{L}$ and $\rho \in (0; 1)$ is the probability of multiaction α.

$\mathcal{SL}$ is the set of *all activities*.

The *alphabet* of $(\alpha, \rho) \in \mathcal{SL}$ is $\mathcal{A}(\alpha, \rho) = \mathcal{A}(\alpha)$.

The operations: *sequential execution* ;, *choice* $[]$, *parallelism* $\|$, *relabeling* $[f]$, *restriction* rs, *synchronization* sy and *iteration* $[**]$.

Relabeling functions $f : \mathcal{A} \to \mathcal{A}$ are bijections preserving conjugates: $\forall x \in \mathcal{A}\ f(\hat{x}) = \widehat{f(x)}$.

Let $\alpha, \beta \in \mathcal{L}$ be two multiactions s.t. for $a \in Act$ we have $a \in \alpha$ and $\hat{a} \in \beta$ or $\hat{a} \in \alpha$ and $a \in \beta$. Then synchronization of α and β by a is $\alpha \oplus_a \beta = \gamma$:

$$\gamma(x) = \begin{cases} \alpha(x) + \beta(x) - 1, & x = a \text{ or } x = \hat{a}; \\ \alpha(x) + \beta(x), & \text{otherwise.} \end{cases}$$

Static expressions specify the structure of processes.

Definition 1 *Let* $(\alpha, \rho) \in \mathcal{SL}$ *and* $a \in Act$. *A* static expression *of* $dtsPBC$ *is*

$$E ::= \ (\alpha, \rho) \mid E;E \mid E[]E \mid E\|E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E{*}E{*}E].$$

$StatExpr$ is the set of *all static expressions* of $dtsPBC$.

Definition 2 *Let* $(\alpha, \rho) \in \mathcal{SL}$ *and* $a \in Act$. *A* regular static expression *of* $dtsPBC$ *is*

$$D ::= \ (\alpha, \rho) \mid D;E \mid D[]D \mid D[f] \mid D \text{ rs } a \mid D \text{ sy } a \mid [D{*}D{*}E],$$
$$E ::= \ (\alpha, \rho) \mid E;E \mid E[]E \mid E\|E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E{*}D{*}E].$$

$RegStatExpr$ is the set of *all regular static expressions* of $dtsPBC$.

Dynamic expressions specify the states of processes.

Definition 3 *Let $(\alpha,\rho)\in\mathcal{SL},\ a\in Act$ and $E\in RegStatExpr$. A regular dynamic expression of $dtsPBC$ is*

$$G ::= \overline{E} \mid \underline{E} \mid G;E \mid E;G \mid G[]E \mid E[]G \mid G\|G \mid G[f] \mid G \text{ rs } a \mid G \text{ sy } a \mid [G{*}E{*}E] \mid [E{*}G{*}E] \mid [E{*}E{*}G].$$

$RegDynExpr$ is the set of *all regular dynamic expressions* of $dtsPBC$.

We shall consider regular expressions only and omit the word “regular”.

Operational semantics

Inaction rules

Inaction rules for overlined and underlined static expressions.

Let $E, F, K \in RegStatExpr$ and $a \in Act$.

$$
\begin{array}{lll}
\overline{E;F} \xrightarrow{\emptyset} \overline{E};F & \underline{E};F \xrightarrow{\emptyset} E;\overline{F} & E;\underline{F} \xrightarrow{\emptyset} \underline{E;F} \\
\overline{E[]F} \xrightarrow{\emptyset} \overline{E}[]F & \overline{E[]F} \xrightarrow{\emptyset} E[]\overline{F} & \underline{E}[]F \xrightarrow{\emptyset} \underline{E[]F} \\
E[]\underline{F} \xrightarrow{\emptyset} \underline{E[]F} & \overline{E\|F} \xrightarrow{\emptyset} \overline{E}\|\overline{F} & \underline{E}\|\underline{F} \xrightarrow{\emptyset} \underline{E\|F} \\
\overline{E[f]} \xrightarrow{\emptyset} \overline{E}[f] & \underline{E}[f] \xrightarrow{\emptyset} \underline{E[f]} & \overline{E \text{ rs } a} \xrightarrow{\emptyset} \overline{E} \text{ rs } a \\
\underline{E} \text{ rs } a \xrightarrow{\emptyset} \underline{E \text{ rs } a} & \overline{E \text{ sy } a} \xrightarrow{\emptyset} \overline{E} \text{ sy } a & \underline{E} \text{ sy } a \xrightarrow{\emptyset} \underline{E \text{ sy } a} \\
\overline{[E*F*K]} \xrightarrow{\emptyset} [\overline{E}*F*K] & [\underline{E}*F*K] \xrightarrow{\emptyset} [E*\overline{F}*K] & [E*\underline{F}*K] \xrightarrow{\emptyset} [E*\overline{F}*K] \\
[E*\underline{F}*K] \xrightarrow{\emptyset} [E*F*\overline{K}] & [E*F*\underline{K}] \xrightarrow{\emptyset} \underline{[E*F*K]} &
\end{array}
$$

Inaction rules for arbitrary dynamic expressions.

Let $E, F \in RegStatExpr,\ G, H, \widetilde{G}, \widetilde{H} \in RegDynExpr$ and $a \in Act$.

$$G \stackrel{\emptyset}{\rightarrow} G \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G},\ \circ\in\{;,[]\}}{G\circ E\stackrel{\emptyset}{\rightarrow}\widetilde{G}\circ E} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G},\ \circ\in\{;,[]\}}{E\circ G\stackrel{\emptyset}{\rightarrow}E\circ\widetilde{G}} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G}}{G\|H\stackrel{\emptyset}{\rightarrow}\widetilde{G}\|H}$$

$$\frac{H\stackrel{\emptyset}{\rightarrow}\widetilde{H}}{G\|H\stackrel{\emptyset}{\rightarrow}G\|\widetilde{H}} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G}}{G[f]\stackrel{\emptyset}{\rightarrow}\widetilde{G}[f]} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G},\ \circ\in\{\mathsf{rs},\mathsf{sy}\}}{G\circ a\stackrel{\emptyset}{\rightarrow}\widetilde{G}\circ a} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G}}{[G*E*F]\stackrel{\emptyset}{\rightarrow}[\widetilde{G}*E*F]}$$

$$\frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G}}{[E*G*F]\stackrel{\emptyset}{\rightarrow}[E*\widetilde{G}*F]} \qquad \frac{G\stackrel{\emptyset}{\rightarrow}\widetilde{G}}{[E*F*G]\stackrel{\emptyset}{\rightarrow}[E*F*\widetilde{G}]}$$

A regular dynamic expression G is *operative* if no inaction rule can be applied to it, with the exception of $G \stackrel{\emptyset}{\rightarrow} G$.

$OpRegDynExpr$ is the set of *all operative regular dynamic expressions* of $dtsPBC$.

Definition 4 $\simeq\ = (\stackrel{\emptyset}{\rightarrow} \cup \stackrel{\emptyset}{\leftarrow})^*$ *is the dynamic expression isomorphism in* $dtsPBC$*.*
G *and* G' *are* isomorphic, $G\simeq G'$, *if they can be reached each from other by applying inaction rules.*

Action rules

Action rules: execution of multisets of activities.

Let $(\alpha,\rho),(\beta,\chi)\in\mathcal{SL},\ E,F\in RegStatExpr,\ G,H\in OpRegDynExpr,$
$\widetilde{G},\widetilde{H}\in RegDynExpr,\ a\in Act$ and $\Gamma,\Delta\in I\!N_f^{\mathcal{SL}}$.

The *alphabet* of $\Gamma\in I\!N_f^{\mathcal{SL}}$ is $\mathcal{A}(\Gamma)=\cup_{(\alpha,\rho)\in\Gamma}\mathcal{A}(\alpha)$.

$$\mathbf{B}\ \ \overline{(\alpha,\rho)}\stackrel{\{(\alpha,\rho)\}}{\longrightarrow}\underline{(\alpha,\rho)}$$

$$\mathbf{SC1}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G},\ \circ\in\{;,[]\}}{G\circ E\stackrel{\Gamma}{\rightarrow}\widetilde{G}\circ E}\qquad \mathbf{SC2}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G},\ \circ\in\{;,[]\}}{E\circ G\stackrel{\Gamma}{\rightarrow}E\circ\widetilde{G}}$$

$$\mathbf{P1}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{G\|H\stackrel{\Gamma}{\rightarrow}\widetilde{G}\|H}\qquad \mathbf{P2}\ \ \frac{H\stackrel{\Gamma}{\rightarrow}\widetilde{H}}{G\|H\stackrel{\Gamma}{\rightarrow}G\|\widetilde{H}}\qquad \mathbf{P3}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G},\ H\stackrel{\Delta}{\rightarrow}\widetilde{H}}{G\|H\stackrel{\Gamma+\Delta}{\longrightarrow}\widetilde{G}\|\widetilde{H}}$$

$$\mathbf{L}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{G[f]\stackrel{f(\Gamma)}{\longrightarrow}\widetilde{G}[f]}\qquad \mathbf{Rs}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G},\ a,\hat{a}\not\in\mathcal{A}(\Gamma)}{G\ \mathsf{rs}\ a\stackrel{\Gamma}{\rightarrow}\widetilde{G}\ \mathsf{rs}\ a}\qquad \mathbf{I1}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{[G*E*F]\stackrel{\Gamma}{\rightarrow}[\widetilde{G}*E*F]}$$

$$\mathbf{I2}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{[E*G*F]\stackrel{\Gamma}{\rightarrow}[E*\widetilde{G}*F]}\qquad \mathbf{I3}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{[E*F*G]\stackrel{\Gamma}{\rightarrow}[E*F*\widetilde{G}]}\qquad \mathbf{Sy1}\ \ \frac{G\stackrel{\Gamma}{\rightarrow}\widetilde{G}}{G\ \mathsf{sy}\ a\stackrel{\Gamma}{\rightarrow}\widetilde{G}\ \mathsf{sy}\ a}$$

$$\mathbf{Sy2}\ \ \frac{G\ \mathsf{sy}\ a\stackrel{\Gamma+\{(\alpha,\rho)\}+\{(\beta,\chi)\}}{\longrightarrow}\widetilde{G}\ \mathsf{sy}\ a,\ a\in\mathcal{A}(\alpha),\ \hat{a}\in\mathcal{A}(\beta)}{G\ \mathsf{sy}\ a\stackrel{\Gamma+\{(\alpha\oplus_a\beta,\rho\cdot\chi)\}}{\longrightarrow}\widetilde{G}\ \mathsf{sy}\ a}$$

Transition systems

$[G]_{\simeq} = \{H \mid G \simeq H\}$ is the isomorphism class of a dynamic expression G.

Definition 5 *The* derivation set *of a dynamic expression G, $DR(G)$, is the minimal set:*

- $[G]_{\simeq} \in DR(G)$;
- *if* $[H]_{\simeq} \in DR(G)$ *and* $\exists\Gamma\ H \xrightarrow{\Gamma} \widetilde{H}$ *then* $[\widetilde{H}]_{\simeq} \in DR(G)$.

Let G be a dynamic expression and $s \in DR(G)$. The set of *all multisets of activities executable from s* is $Exec(s) = \{\Gamma \mid \exists H \in s\ \exists\widetilde{H}\ H \xrightarrow{\Gamma} \widetilde{H}\}$.

The probability that the activities from $\Gamma \in Exec(s)$ *try to happen* in s is

$$PF(\Gamma, s) = \prod_{(\alpha,\rho)\in\Gamma} \rho \cdot \prod_{\{\{(\beta,\chi)\}\in Exec(s) \mid (\beta,\chi)\not\in\Gamma\}} (1-\chi).$$

In the case $\Gamma = \emptyset$ we define

$$PF(\emptyset, s) = \begin{cases} \prod_{\{(\beta,\chi)\}\in Exec(s)} (1-\chi), & Exec(s) \neq \{\emptyset\}; \\ 1, & \text{otherwise.} \end{cases}$$

The probability that the activities from Γ *happen* in s is

$$PT(\Gamma, s) = \frac{PF(\Gamma, s)}{\sum_{\Delta \in Exec(s)} PF(\Delta, s)}.$$

The probability that the execution of *any* activities changes s by $\tilde{s}$ is

$$PM(s, \tilde{s}) = \sum_{\{\Gamma \mid \exists H \in s\ \exists \widetilde{H} \in \tilde{s}\ H \xrightarrow{\Gamma} \widetilde{H}\}} PT(\Gamma, s).$$

Definition 6 *The* (labeled probabilistic) transition system *of a dynamic expression* G *is* $TS(G) = (S_G, L_G, \mathcal{T}_G, s_G)$, *where*

- *the set of* states *is* $S_G = DR(G)$;
- *the set of* labels *is* $L_G \subseteq I\!N_f^{\mathcal{SL}} \times (0; 1]$;
- *the set of* transitions *is* $\mathcal{T}_G = \{(s, (\Gamma, PT(\Gamma, s)), \tilde{s}) \mid s \in DR(G),\ \exists H \in s\ \exists \widetilde{H} \in \tilde{s}\ H \stackrel{\Gamma}{\rightarrow} \widetilde{H}\}$;
- *the* initial state *is* $s_G = [G]_{\simeq}$.

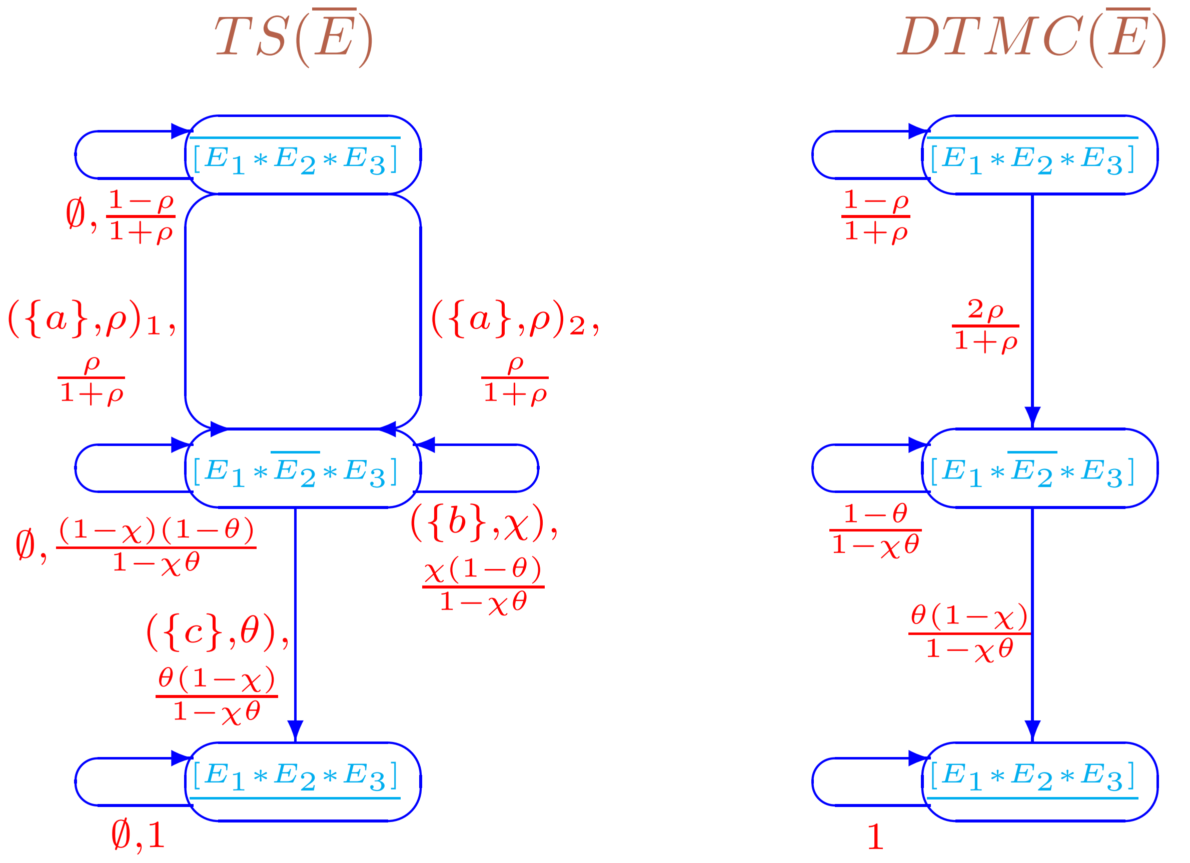
A transition $(s, (\Gamma, \mathcal{P}), \tilde{s}) \in \mathcal{T}_G$ is written as $s \stackrel{\Gamma}{\rightarrow}_{\mathcal{P}} \tilde{s}$.

We write $s \stackrel{\Gamma}{\rightarrow} \tilde{s}$ if $\exists \mathcal{P}\ s \stackrel{\Gamma}{\rightarrow}_{\mathcal{P}} \tilde{s}$ and $s \rightarrow \tilde{s}$ if $\exists \Gamma\ s \stackrel{\Gamma}{\rightarrow} \tilde{s}$.

We denote *isomorphism of transition systems* by $\simeq$.

Definition 7 G *and* G' *are* isomorphic w.r.t. transition systems, $G =_{ts} G'$, *if* $TS(G) \simeq TS(G')$.

Definition 8 *The* underlying discrete time Markov chain (DTMC) *of a dynamic expression* G, $DTMC(G)$, *has the state space* $DR(G)$ *and transitions* $s \rightarrow_{PM(s,\tilde{s})} \tilde{s}$, *if* $s \rightarrow \tilde{s}$.



EXPRIT:The transition system and the underlying DTMC of $\overline{E}$ for

$$E = [((\{a\},\rho)_1[](\{a\},\rho)_2) * (\{b\},\chi) * (\{c\},\theta)]$$

Let $E_1 = (\{a\},\rho)[](\{a\},\rho)$, $E_2 = (\{b\},\chi)$, $E_3 = (\{c\},\theta)$ and $E = [E_1 * E_2 * E_3]$.

The identical activities of the composite static expression are enumerated as:

$E = [((\{a\},\rho)_1[](\{a\},\rho)_2) * (\{b\},\chi) * (\{c\},\theta)]$.

Denotational semantics

Definition 9 *A* plain discrete time stochastic Petri box (plain dts-box) *is* $N = (P_N, T_N, W_N, \Lambda_N)$*:*

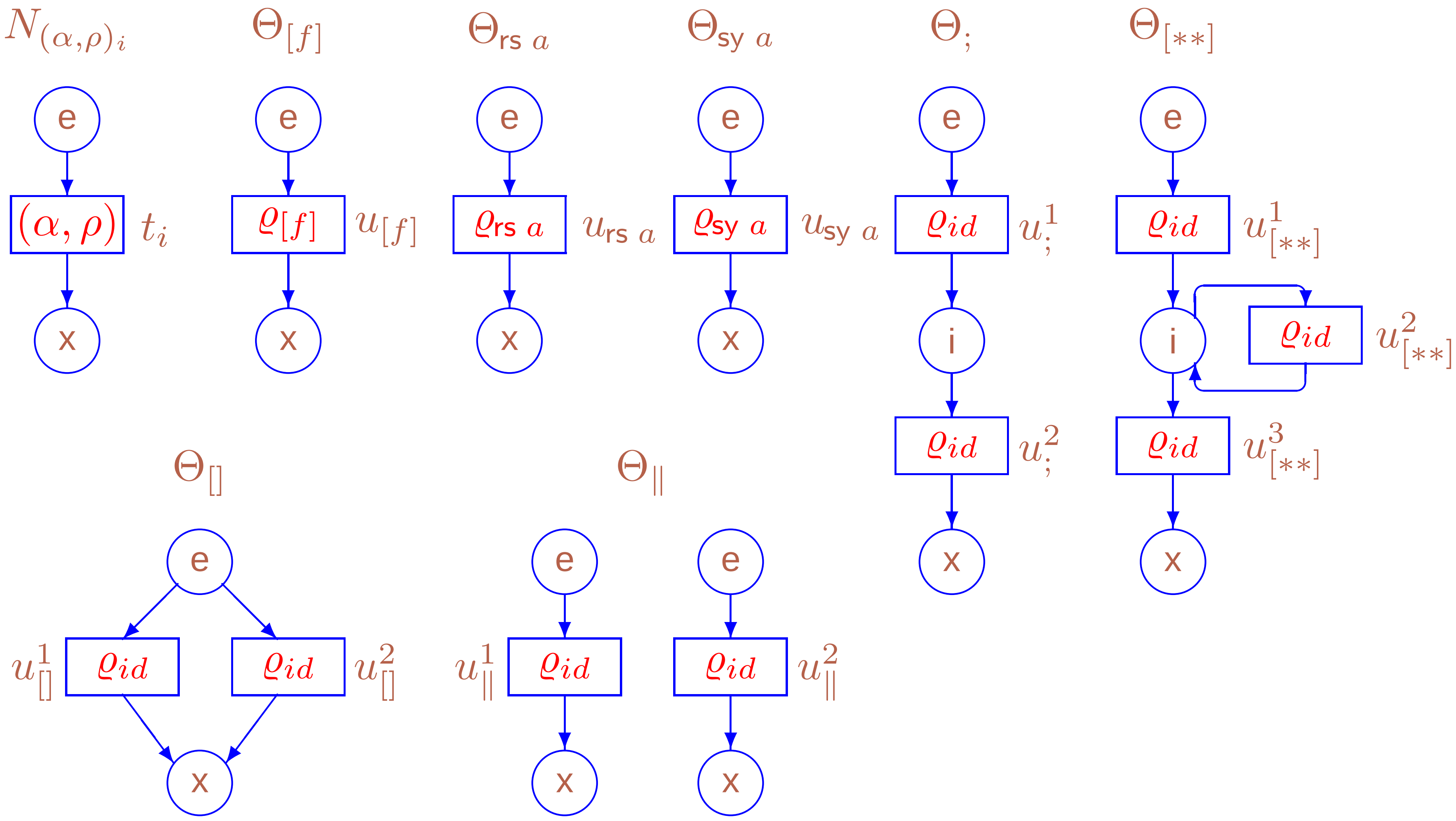
- P_N *and* T_N *are finite sets of* places *and* transitions*, respectively, s.t.* $P_N \cup T_N \neq \emptyset$ *and* $P_N \cap T_N = \emptyset$*;*
- $W_N : (P_N \times T_N) \cup (T_N \times P_N) \to \mathbb{N}$ *is a function of the* weights of arcs *between places and transitions and vice versa;*
- Λ_N *is the* place and transition labeling *function s.t.*
  $\Lambda_N : P_N \to \{\mathsf{e}, \mathsf{i}, \mathsf{x}\}$ *(it specifies* entry, internal *and* exit *places) and*
  $\Lambda_N : T_N \to \mathcal{SL}$ *(it associates* activities *with transitions).*

Moreover, $\forall t \in T_N\ {}^\bullet t \neq \emptyset \neq t^\bullet$*.*

For the set of entry *places of* N*,* ${}^\circ N = \{p \in P_N \mid \Lambda_N(p) = \mathsf{e}\}$*, and the set of* exit *places of* N*,* $N^\circ = \{p \in P_N \mid \Lambda_N(p) = \mathsf{x}\}$*, it holds:* ${}^\circ N \neq \emptyset \neq N^\circ$ *and* ${}^\bullet({}^\circ N) = \emptyset = (N^\circ)^\bullet$*.*

A *marked plain dts-box* is a pair (N, M_N), where N is a plain dts-box and $M_N \in \mathbb{N}_f^{P_N}$ is the *initial marking*. Let $\overline{N} = (N, {}^\circ N)$ and $\underline{N} = (N, N^\circ)$.

A marked plain dts-box $(P_N, T_N, W_N, \Lambda_N, M_N)$ is the LDTSPN $(P_N, T_N, W_N, \Omega_N, L_N, M_N)$, where $\forall t \in T_N\ \Omega_N(t) = \Omega(\Lambda_N(t))$ (*transition probability*), $L_N(t) = \mathcal{L}(\Lambda_N(t))$ (*transition labeling*).



The plain and operator dts-boxes

Definition 10 *Let* $(\alpha, \rho) \in \mathcal{SL},\ a \in Act$ *and* $E, F, K \in RegStatExpr$. *The* denotational semantics *of* $dtsPBC$ *is a mapping* Box_{dts} *from* $RegStatExpr$ *into plain dts-boxes:*

1. $Box_{dts}((\alpha, \rho)_i) = N_{(\alpha,\rho)_i}$;
2. $Box_{dts}(E \circ F) = \Theta_{\circ}(Box_{dts}(E), Box_{dts}(F)),\ \circ \in \{;, [], \|\}$;
3. $Box_{dts}(E[f]) = \Theta_{[f]}(Box_{dts}(E))$;
4. $Box_{dts}(E \circ a) = \Theta_{\circ a}(Box_{dts}(E)),\ \circ \in \{\mathsf{rs}, \mathsf{sy}\}$;
5. $Box_{dts}([E * F * K]) = \Theta_{[**]}(Box_{dts}(E), Box_{dts}(F), Box_{dts}(K))$.

For $E \in RegStatExpr$, let $Box_{dts}(\overline{E}) = \overline{Box_{dts}(E)}$ and $Box_{dts}(\underline{E}) = \underline{Box_{dts}(E)}$.

We denote isomorphism of transition systems by $\simeq$,

and the same symbol denotes isomorphism of reachability graphs and DTMCs

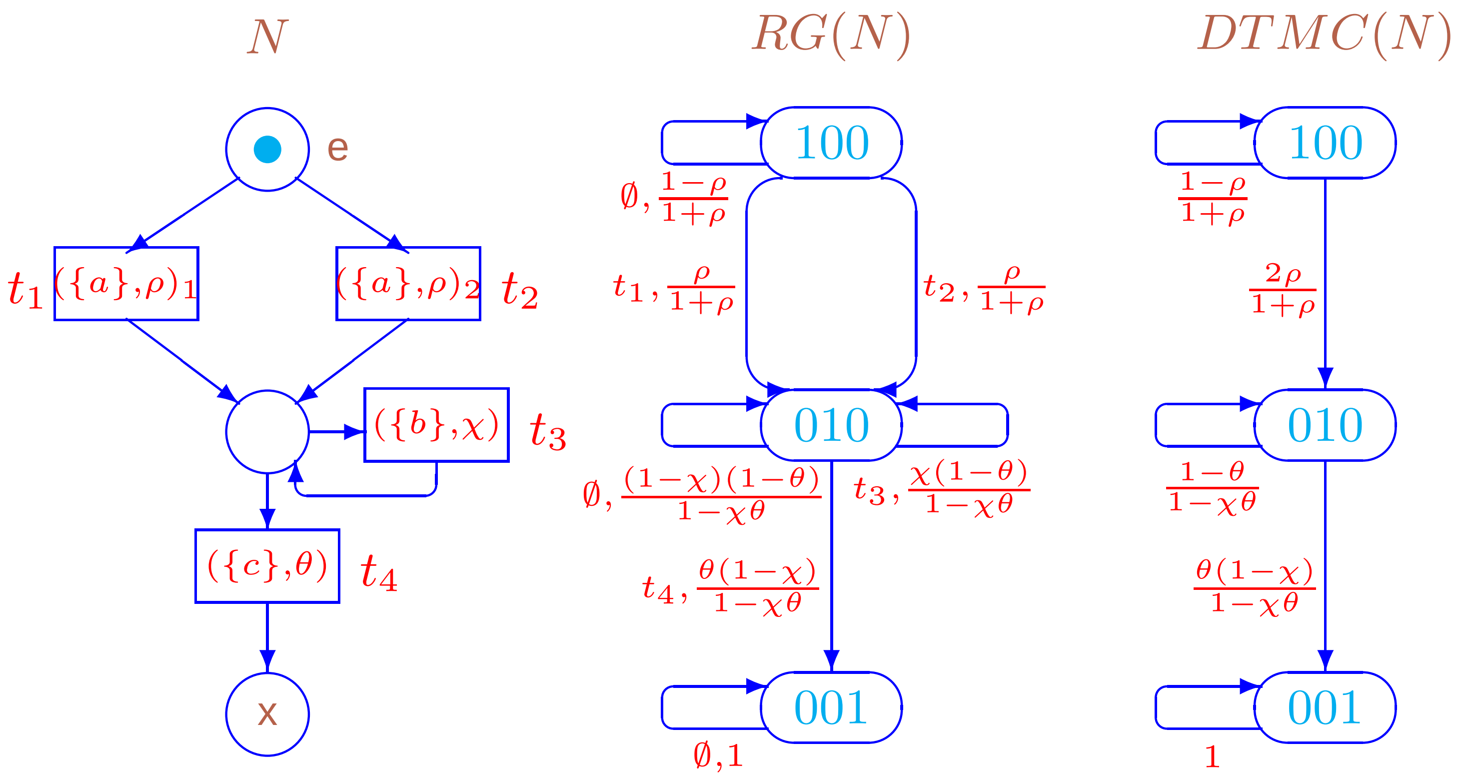
as well as isomorphism between transition systems and reachability graphs.

Theorem 1 *For any static expression* E

$$TS(\overline{E}) \simeq RG(Box_{dts}(\overline{E})).$$

Proposition 1 *For any static expression* E

$$DTMC(\overline{E}) \simeq DTMC(Box_{dts}(\overline{E})).$$



BOXIT:The marked dts-box $N = Box_{dts}(\overline{E})$ for $E = [((\{a\},\rho)_1[](\{a\},\rho)_2) * (\{b\},\chi) * (\{c\},\theta)]$, its reachability graph and the underlying DTMC

Performance evaluation

Empty loops

Let G be a dynamic expression. The *probability to execute in $s \in DR(G)$ a non-empty multiset of activities $\Gamma \in Exec(s) \setminus \{\emptyset\}$ after possible empty loops* is

$$PT^*(\Gamma, s) = PT(\Gamma, s) \cdot \sum_{k=0}^{\infty} (PT(\emptyset, s))^k = \frac{PT(\Gamma, s)}{1 - PT(\emptyset, s)} = SJ(s) \cdot PT(\Gamma, s).$$

Definition 11 *The* (labeled probabilistic) transition system without empty loops $TS^*(G)$ *has the state space $DR(G)$ and the transitions $s \stackrel{\Gamma}{\twoheadrightarrow}_{PT^*(\Gamma,s)} \tilde{s}$, if $s \stackrel{\Gamma}{\rightarrow} \tilde{s},\ \Gamma \neq \emptyset$.*

We write $s \stackrel{\Gamma}{\twoheadrightarrow} \tilde{s}$ if $\exists \mathcal{P}\ s \stackrel{\Gamma}{\twoheadrightarrow}_{\mathcal{P}} \tilde{s}$ and $s \twoheadrightarrow \tilde{s}$ if $\exists \Gamma\ s \stackrel{\Gamma}{\twoheadrightarrow} \tilde{s}$.

Definition 12 *G and G' are* isomorphic w.r.t. transition systems without empty loops, *$G =_{ts*} G'$, if $TS^*(G) \simeq TS^*(G')$.*

Definition 13 *The* underlying DTMC without empty loops $DTMC^*(G)$ *has the state space* $DR(G)$ *and transitions* $s \twoheadrightarrow_{PM^*(s,\tilde{s})} \tilde{s}$, *if* $s \twoheadrightarrow \tilde{s}$, *where*

$$PM^*(s,\tilde{s}) = \sum_{\{\Gamma | s \overset{\Gamma}{\twoheadrightarrow} \tilde{s}\}} PT^*(\Gamma, s).$$

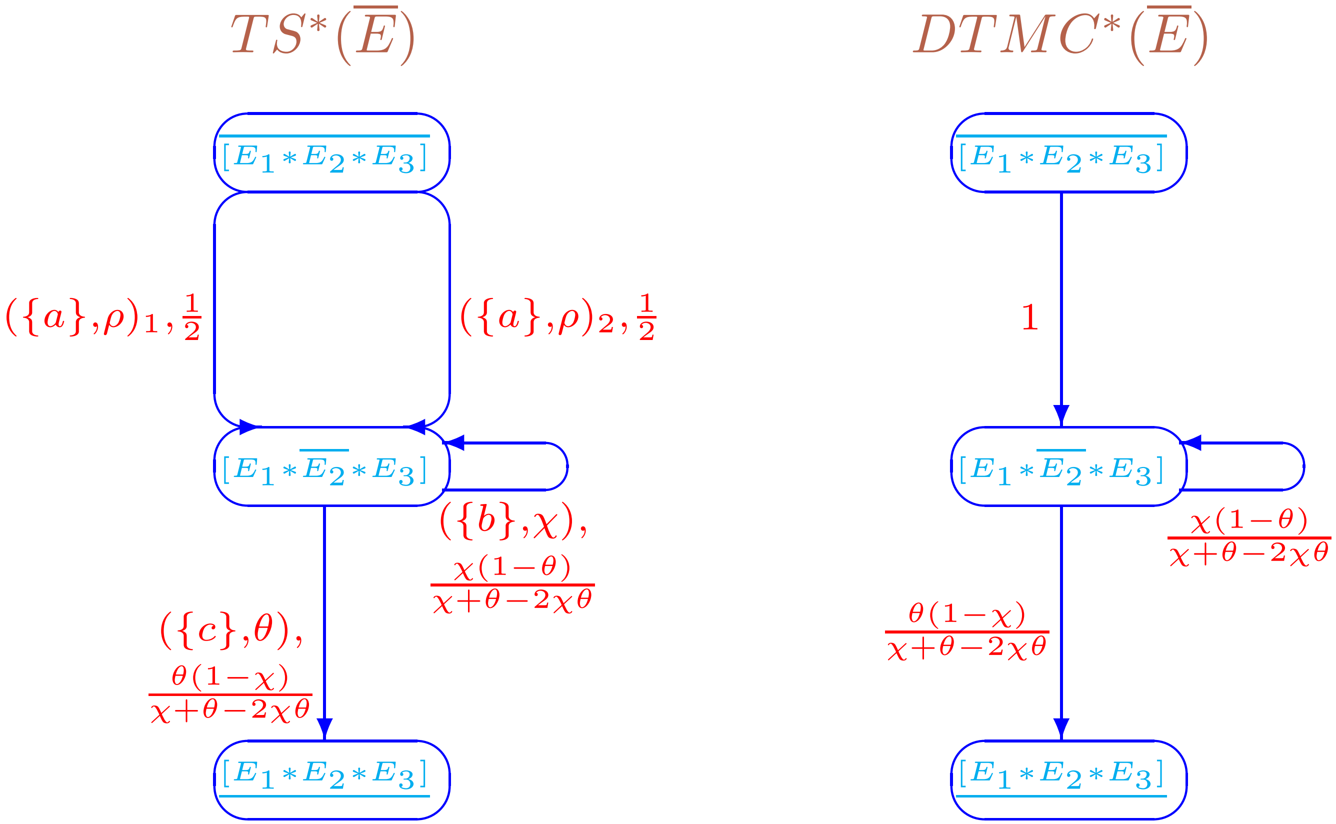
Let $N = (P_N, T_N, W_N, \Omega_N, L_N, M_N)$ be a LDTSPN and $M, \widetilde{M} \in \mathbb{N}_f^{P_N}$, $t \in T_N$, $U \subseteq T_N$. Then the transition relations $M \overset{U}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$, $M \overset{U}{\twoheadrightarrow} \widetilde{M}$, $M \twoheadrightarrow \widetilde{M}$, $M \twoheadrightarrow_{\mathcal{P}} \widetilde{M}$, the *reachability graph without empty loops* $RG^*(N)$ and the *underlying DTMC without empty loops* $DTMC^*(N)$ are defined like for dynamic expressions.

Theorem 2 *For any static expression* E

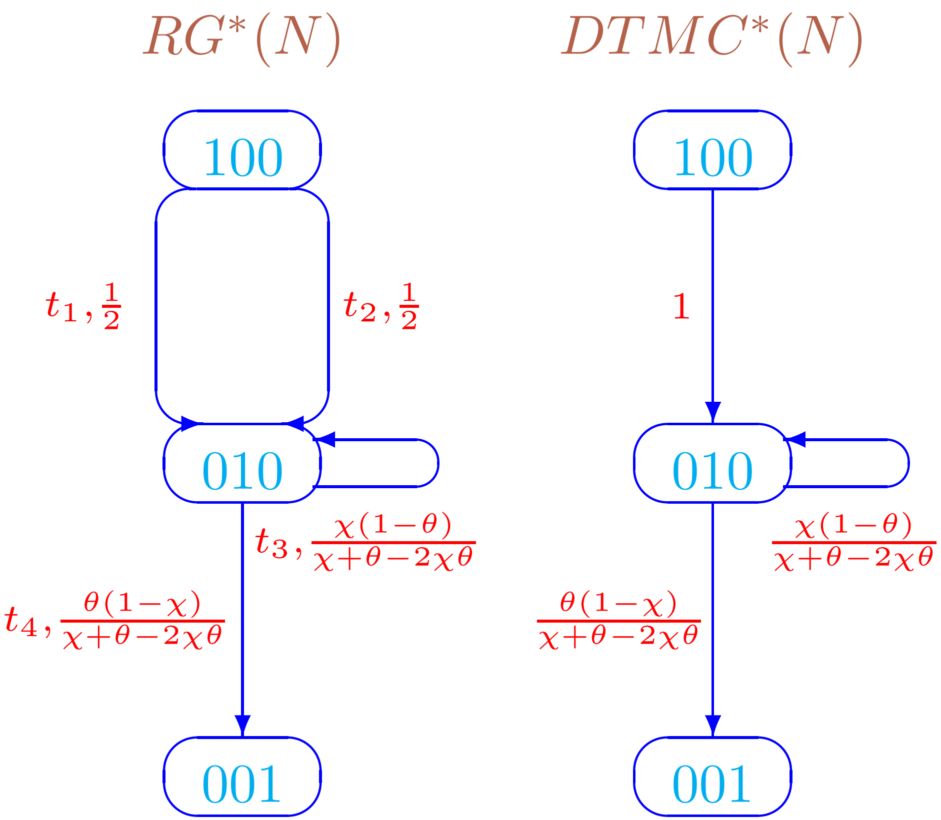
$$TS^*(\overline{E}) \simeq RG^*(Box_{dts}(\overline{E})).$$

Proposition 2 *For any static expression* E

$$DTMC^*(\overline{E}) \simeq DTMC^*(Box_{dts}(\overline{E})).$$



The transition system and the underlying DTMC without empty loops of $\overline{E}$ from Figure EXPRIT



The reachability graph and the underlying DTMC without empty loops of N from Figure BOXIT

Stationary behaviour

The elements $\mathcal{P}^*_{ij}$ $(1 \le i, j \le n = |DR(G)|)$ of *(one-step) transition probability matrix (TPM)* $\mathbf{P}^*$ for $DTMC^*(G)$:

$$\mathcal{P}^*_{ij} = \begin{cases} PM^*(s_i, s_j), & s_i \to s_j; \\ 0, & \text{otherwise.} \end{cases}$$

The *transient (k-step, $k \in I\!N$) probability mass function (PMF)* $\psi^*[k] = (\psi^*_1[k], \ldots, \psi^*_n[k])$ for $DTMC^*(N)$ is the solution of

$$\psi^*[k] = \psi^*[0](\mathbf{P}^*)^k,$$

where $\psi^*[0] = (\psi^*_1[0], \ldots, \psi^*_n[0])$ is the *initial PMF*:

$$\psi^*_i[0] = \begin{cases} 1, & s_i = [G]_\simeq; \\ 0, & \text{otherwise.} \end{cases}$$

We have $\psi^*[k+1] = \psi^*[k]\mathbf{P}^*,\ k \in I\!N$.

The *steady state PMF* $\psi^* = (\psi_1^*, \ldots, \psi_n^*)$ for $DTMC^*(G)$ is the solution of

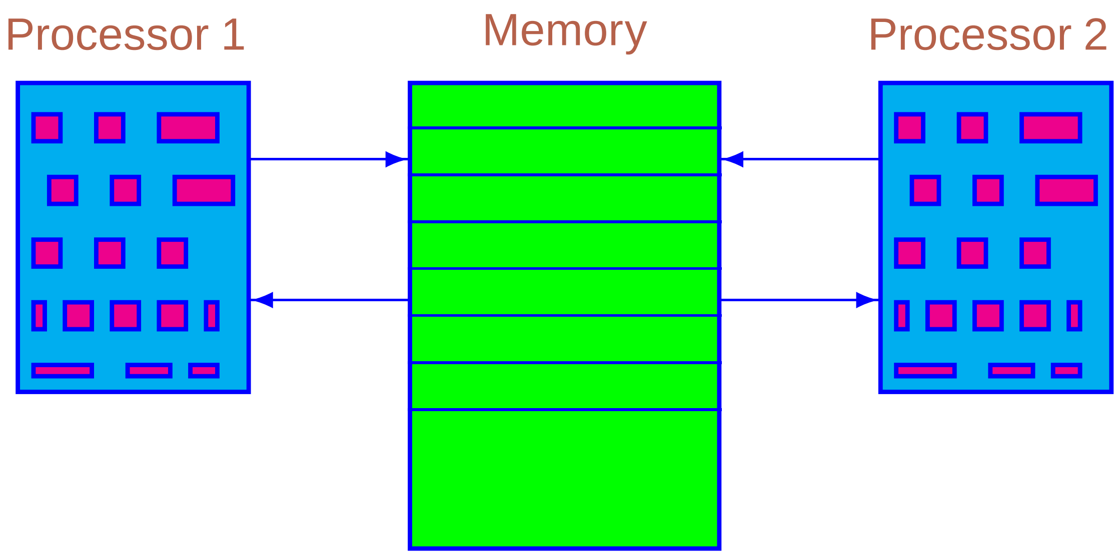
$$\begin{cases} \psi^*(\mathbf{P}^* - \mathbf{E}) = \mathbf{0} \\ \psi^*\mathbf{1}^T = 1 \end{cases},$$

where $\mathbf{0}$ is a vector with n values $0,\ \mathbf{1}$ is that with n values 1.

When $DTMC^*(G)$ has the steady state, $\psi^* = \lim_{k\to\infty} \psi^*[k]$.

Shared memory system

A model of two processors accessing a common shared memory [MBCDF95]



The diagram of the shared memory system

After activation of the system, two processors are active, and the common memory is available. Each processor can request an access to the memory.

When a processor starts an acquisition of the memory, another processor waits until the former one ends its memory operations, and the system returns to the state with both active processors and the available common memory.

a corresponds to the system activation.

r_i $(1 \leq i \leq 2)$ represent the common memory request of processor i.

b_i and e_i correspond to the beginning and the end of the common memory access of processor i.

The other actions are used for communication purpose only.

$\mathsf{Stop} = (\{c\}, \frac{1}{2})$ rs c is the process that performs empty loops with probability 1 and never terminates.

The static expression of the first processor is

$E_1 = [(\{x_1\}, \frac{1}{2}) * ((\{r_1\}, \frac{1}{2}); (\{b_1, y_1\}, \frac{1}{2}); (\{e_1, z_1\}, \frac{1}{2})) * \mathsf{Stop}]$.

The static expression of the second processor is

$E_2 = [(\{x_2\}, \frac{1}{2}) * ((\{r_2\}, \frac{1}{2}); (\{b_2, y_2\}, \frac{1}{2}); (\{e_2, z_2\}, \frac{1}{2})) * \mathsf{Stop}]$.

The static expression of the shared memory is

$E_3 = [(\{a, \widehat{x_1}, \widehat{x_2}\}, \frac{1}{2}) * (((\{\widehat{y_1}\}, \frac{1}{2}); (\{\widehat{z_1}\}, \frac{1}{2}))[]((\{\widehat{y_2}\}, \frac{1}{2}); (\{\widehat{z_2}\}, \frac{1}{2}))) * \mathsf{Stop}]$.

The static expression of the shared memory system with two processors is

$E = (E_1 \| E_2 \| E_3)$ sy x_1 sy x_2 sy y_1 sy y_2 sy z_1 sy z_2 rs x_1 rs x_2 rs y_1 rs y_2 rs z_1 rs z_2.

$DR(\overline{E})$ consists of isomorphism classes

$$s_1 = [([\overline{(\{x_1\}, \tfrac{1}{2})} * ((\{r_1\}, \tfrac{1}{2}); (\{b_1, y_1\}, \tfrac{1}{2}); (\{e_1, z_1\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [\overline{(\{x_2\}, \tfrac{1}{2})} * ((\{r_2\}, \tfrac{1}{2}); (\{b_2, y_2\}, \tfrac{1}{2}); (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [\overline{(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2})} * (((\{\widehat{y_1}\}, \tfrac{1}{2}); (\{\widehat{z_1}\}, \tfrac{1}{2}))[]((\{\widehat{y_2}\}, \tfrac{1}{2}); (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

$$s_2 = [([(\{x_1\}, \tfrac{1}{2}) * (\overline{(\{r_1\}, \tfrac{1}{2})}; (\{b_1, y_1\}, \tfrac{1}{2}); (\{e_1, z_1\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [(\{x_2\}, \tfrac{1}{2}) * (\overline{(\{r_2\}, \tfrac{1}{2})}; (\{b_2, y_2\}, \tfrac{1}{2}); (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2}) * (((\{\widehat{y_1}\}, \tfrac{1}{2}); (\{\widehat{z_1}\}, \tfrac{1}{2}))[]((\{\widehat{y_2}\}, \tfrac{1}{2}); (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

$$s_3 = [([(\{x_1\}, \tfrac{1}{2}) * ((\{r_1\}, \tfrac{1}{2}); \overline{(\{b_1, y_1\}, \tfrac{1}{2})}; (\{e_1, z_1\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [(\{x_2\}, \tfrac{1}{2}) * (\overline{(\{r_2\}, \tfrac{1}{2})}; (\{b_2, y_2\}, \tfrac{1}{2}); (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\| [(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2}) * ((\overline{(\{\widehat{y_1}\}, \tfrac{1}{2})}; (\{\widehat{z_1}\}, \tfrac{1}{2}))[]((\{\widehat{y_2}\}, \tfrac{1}{2}); (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

$$s_4 = [([(\{x_1\}, \tfrac{1}{2}) * (\overline{(\{r_1\}, \tfrac{1}{2})}; (\{b_1, y_1\}, \tfrac{1}{2}); (\{e_1, z_1\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\|[(\{x_2\}, \tfrac{1}{2}) * ((\{r_2\}, \tfrac{1}{2}); \overline{(\{b_2, y_2\}, \tfrac{1}{2})}; (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\|[(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2}) * (((\{\widehat{y_1}\}, \tfrac{1}{2}); (\{\widehat{z_1}\}, \tfrac{1}{2}))[](\overline{(\{\widehat{y_2}\}, \tfrac{1}{2})}; (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

$$s_5 = [([(\{x_1\}, \tfrac{1}{2}) * ((\{r_1\}, \tfrac{1}{2}); (\{b_1, y_1\}, \tfrac{1}{2}); \overline{(\{e_1, z_1\}, \tfrac{1}{2})}) * \mathsf{Stop}]$$
$$\|[(\{x_2\}, \tfrac{1}{2}) * (\overline{(\{r_2\}, \tfrac{1}{2})}; (\{b_2, y_2\}, \tfrac{1}{2}); (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\|[(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2}) * (((\{\widehat{y_1}\}, \tfrac{1}{2}); \overline{(\{\widehat{z_1}\}, \tfrac{1}{2})})[]((\{\widehat{y_2}\}, \tfrac{1}{2}); (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

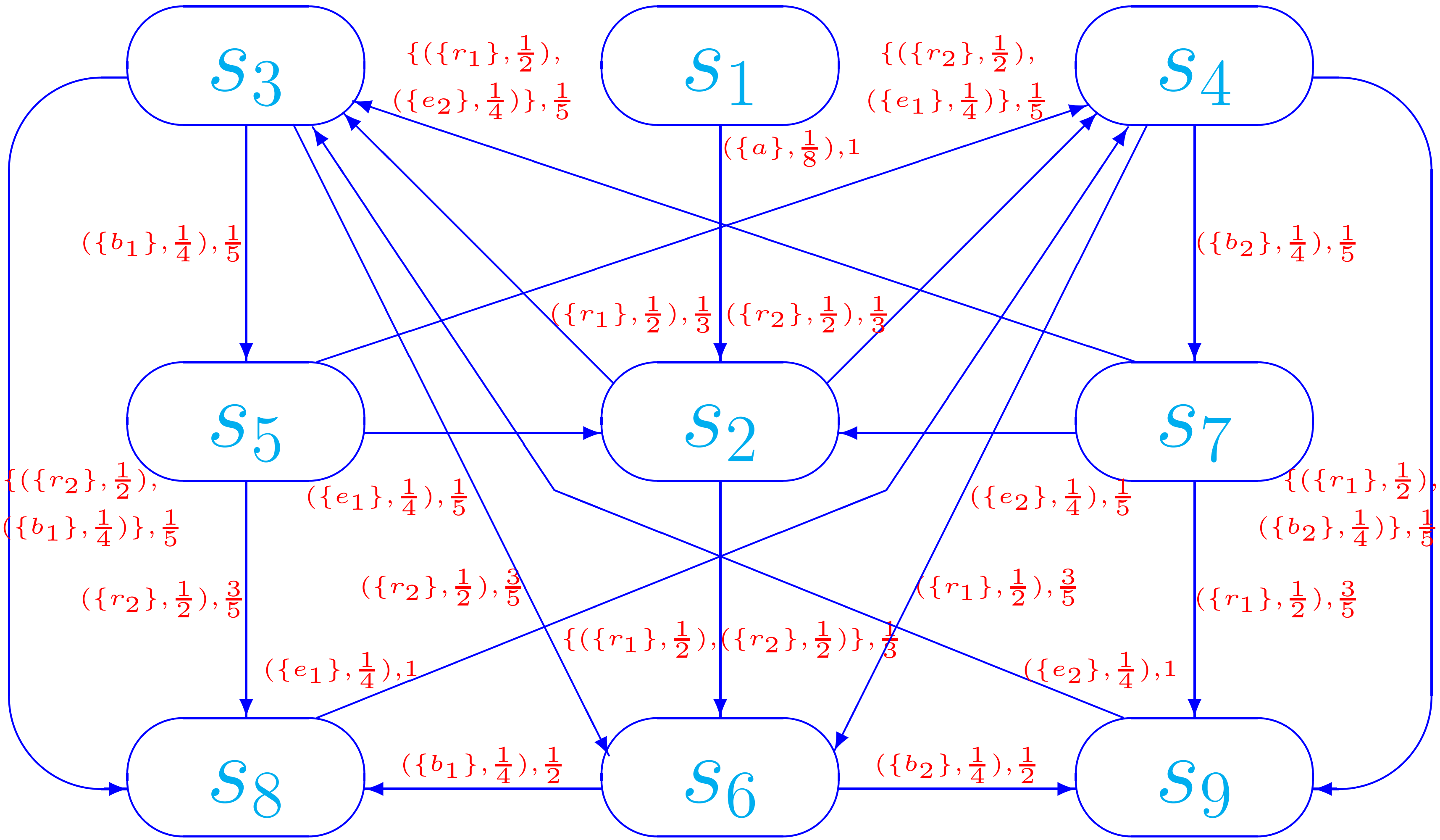
$$s_6 = [([(\{x_1\}, \tfrac{1}{2}) * ((\{r_1\}, \tfrac{1}{2}); \overline{(\{b_1, y_1\}, \tfrac{1}{2})}; (\{e_1, z_1\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\|[(\{x_2\}, \tfrac{1}{2}) * ((\{r_2\}, \tfrac{1}{2}); \overline{(\{b_2, y_2\}, \tfrac{1}{2})}; (\{e_2, z_2\}, \tfrac{1}{2})) * \mathsf{Stop}]$$
$$\|[(\{a, \widehat{x_1}, \widehat{x_2}\}, \tfrac{1}{2}) * ((\overline{(\{\widehat{y_1}\}, \tfrac{1}{2})}; (\{\widehat{z_1}\}, \tfrac{1}{2}))[](\overline{(\{\widehat{y_2}\}, \tfrac{1}{2})}; (\{\widehat{z_2}\}, \tfrac{1}{2}))) * \mathsf{Stop}])$$
$$\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},$$

$$
\begin{array}{l}
s_7 = [([(\{x_1\},\tfrac{1}{2}) * ((\overline{(\{r_1\},\tfrac{1}{2})};(\{b_1,y_1\},\tfrac{1}{2});(\{e_1,z_1\},\tfrac{1}{2})) * \mathsf{Stop}] \\
\|[(\{x_2\},\tfrac{1}{2}) * ((\{r_2\},\tfrac{1}{2});(\{b_2,y_2\},\tfrac{1}{2});\overline{(\{e_2,z_2\},\tfrac{1}{2})}) * \mathsf{Stop}] \\
\|[(\{a,\widehat{x_1},\widehat{x_2}\},\tfrac{1}{2}) * (((\{\widehat{y_1}\},\tfrac{1}{2});(\{\widehat{z_1}\},\tfrac{1}{2}))[]((\{\widehat{y_2}\},\tfrac{1}{2});\overline{(\{\widehat{z_2}\},\tfrac{1}{2})})) * \mathsf{Stop}]) \\
\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},
\end{array}
$$

$$
\begin{array}{l}
s_8 = [([(\{x_1\},\tfrac{1}{2}) * ((\{r_1\},\tfrac{1}{2});(\{b_1,y_1\},\tfrac{1}{2});\overline{(\{e_1,z_1\},\tfrac{1}{2})}) * \mathsf{Stop}] \\
\|[(\{x_2\},\tfrac{1}{2}) * ((\{r_2\},\tfrac{1}{2});(\{b_2,y_2\},\tfrac{1}{2});(\{e_2,z_2\},\tfrac{1}{2})) * \mathsf{Stop}] \\
\|[(\{a,\widehat{x_1},\widehat{x_2}\},\tfrac{1}{2}) * (((\{\widehat{y_1}\},\tfrac{1}{2});\overline{(\{\widehat{z_1}\},\tfrac{1}{2})})[]((\{\widehat{y_2}\},\tfrac{1}{2});(\{\widehat{z_2}\},\tfrac{1}{2}))) * \mathsf{Stop}]) \\
\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq},
\end{array}
$$

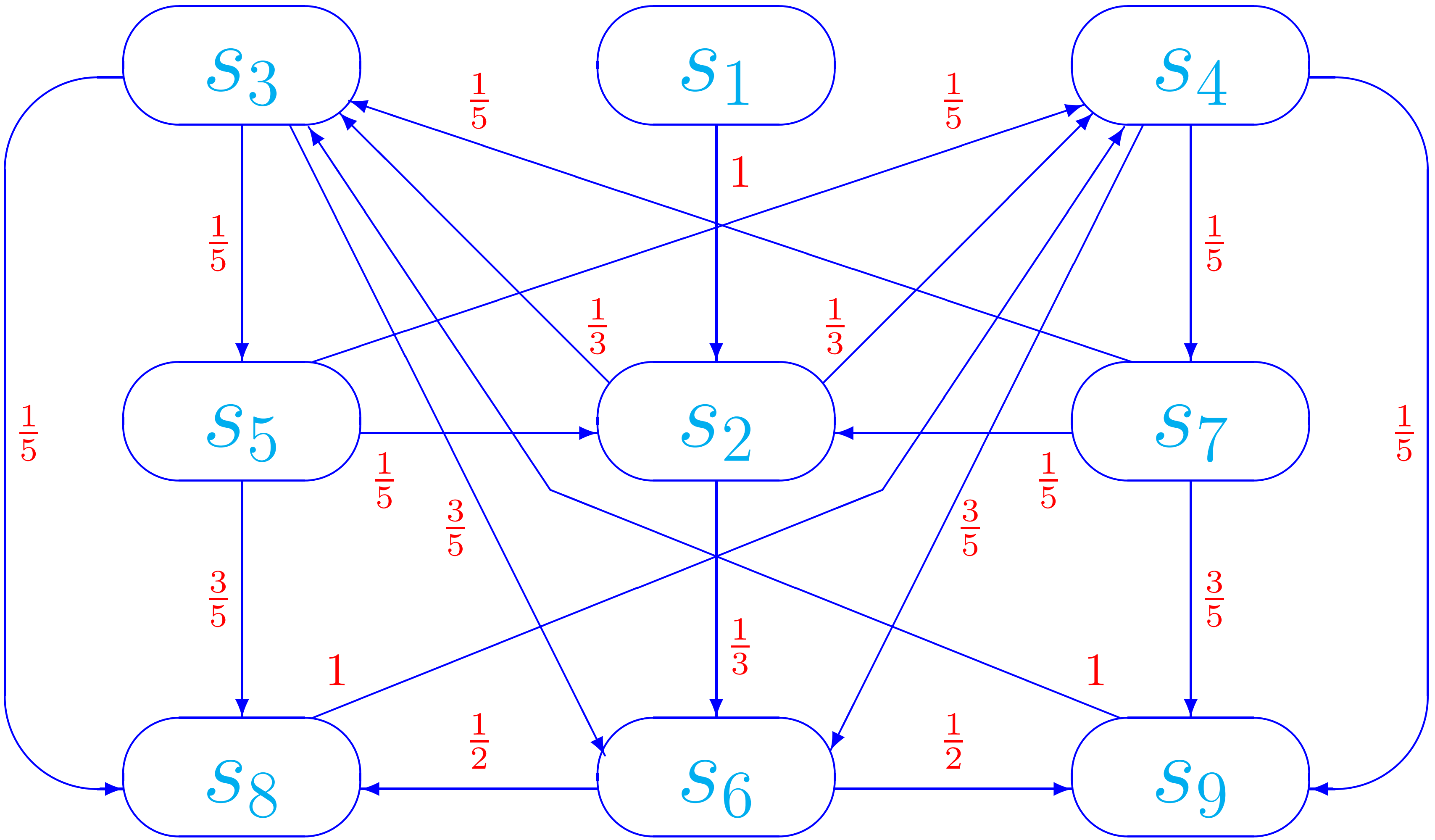
$$
\begin{array}{l}
s_9 = [([(\{x_1\},\tfrac{1}{2}) * ((\{r_1\},\tfrac{1}{2});(\{b_1,y_1\},\tfrac{1}{2});(\{e_1,z_1\},\tfrac{1}{2})) * \mathsf{Stop}] \\
\|[(\{x_2\},\tfrac{1}{2}) * ((\{r_2\},\tfrac{1}{2});(\{b_2,y_2\},\tfrac{1}{2});\overline{(\{e_2,z_2\},\tfrac{1}{2})}) * \mathsf{Stop}] \\
\|[(\{a,\widehat{x_1},\widehat{x_2}\},\tfrac{1}{2}) * (((\{\widehat{y_1}\},\tfrac{1}{2});(\{\widehat{z_1}\},\tfrac{1}{2}))[]((\{\widehat{y_2}\},\tfrac{1}{2});\overline{(\{\widehat{z_2}\},\tfrac{1}{2})})) * \mathsf{Stop}]) \\
\mathsf{sy}\ x_1\ \mathsf{sy}\ x_2\ \mathsf{sy}\ y_1\ \mathsf{sy}\ y_2\ \mathsf{sy}\ z_1\ \mathsf{sy}\ z_2\ \mathsf{rs}\ x_1\ \mathsf{rs}\ x_2\ \mathsf{rs}\ y_1\ \mathsf{rs}\ y_2\ \mathsf{rs}\ z_1\ \mathsf{rs}\ z_2]_{\simeq}.
\end{array}
$$

$TS^*(\overline{E})$



The transition system without empty loops of the shared memory system

$DTMC^*(\overline{E})$



The underlying DTMC without empty loops of the shared memory system

The TPM for $DTMC^*(\overline{E})$ is

$$\mathbf{P}^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & \frac{3}{5} & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\ 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

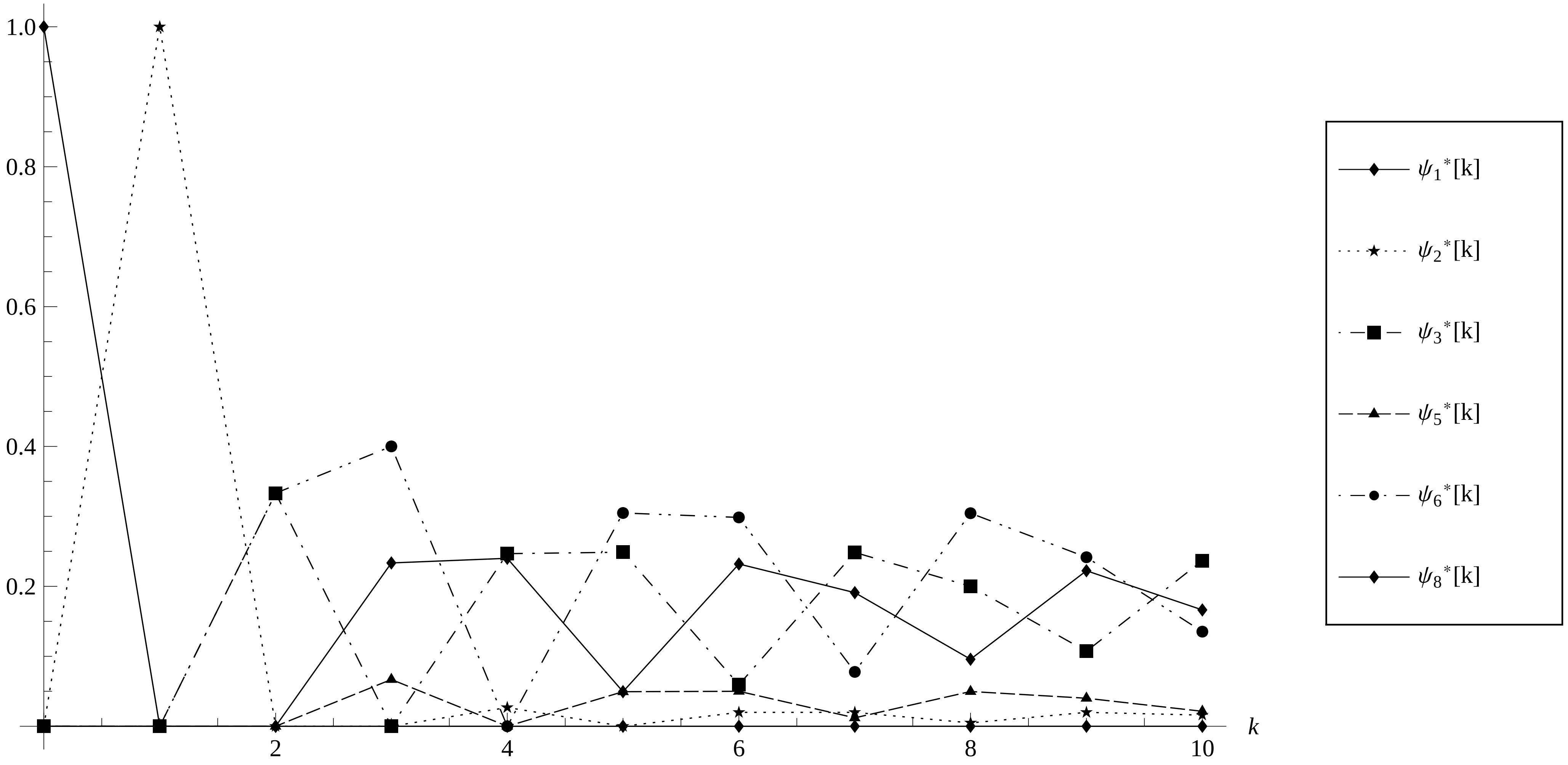
The steady state PMF ψ^* for $DTMC^*(\overline{E})$ is

$$\psi^* = \left(0, \frac{3}{209}, \frac{75}{418}, \frac{75}{418}, \frac{15}{418}, \frac{46}{209}, \frac{15}{418}, \frac{35}{209}, \frac{35}{209}\right).$$

Transient state probabilities of the shared memory system

k	0	1	2	3	4	5	6	7	8	9	10	∞
$\psi_1^*[k]$	1	0	0	0	0	0	0	0	0	0	0	0
$\psi_2^*[k]$	0	1	0	0	0.0267	0	0.0197	0.0199	0.0047	0.0199	0.0160	0.0144
$\psi_3^*[k]$	0	0	0.3333	0	0.2467	0.2489	0.0592	0.2484	0.2000	0.1071	0.2368	0.1794
$\psi_5^*[k]$	0	0	0	0.0667	0	0.0493	0.0498	0.0118	0.0497	0.0400	0.0214	0.0359
$\psi_6^*[k]$	0	0	0.3333	0.4000	0	0.3049	0.2987	0.0776	0.3047	0.2416	0.1351	0.2201
$\psi_8^*[k]$	0	0	0	0.2333	0.2400	0.0493	0.2318	0.1910	0.0956	0.2221	0.1662	0.1675

We depict the probabilities for the states $s_1, s_2, s_3, s_5, s_6, s_8$ only, since the corresponding values coincide for s_3, s_4 as well as for s_5, s_7 as well as for s_8, s_9.



Transient state probabilities alteration diagram of the shared memory system

Performance indices

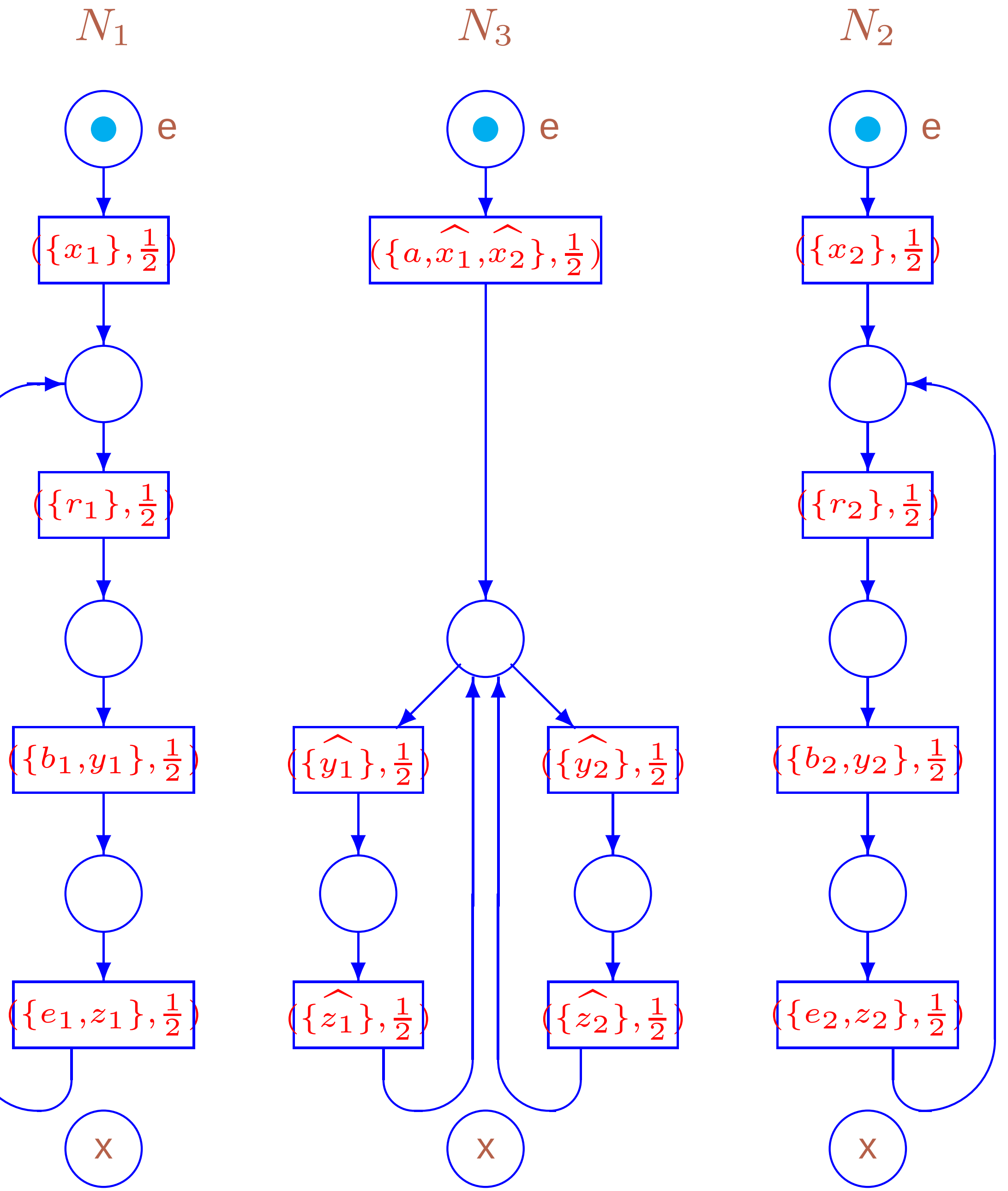
- The average recurrence time in the state s_2, the *average system run-through*, is $\frac{1}{\psi_2^*} = \frac{209}{3} = 69\frac{2}{3}$.
- The common memory is available in the states s_2, s_3, s_4, s_6 only.

  The steady state probability that the memory is available is $\psi_2^* + \psi_3^* + \psi_4^* + \psi_6^* = \frac{124}{209}$.

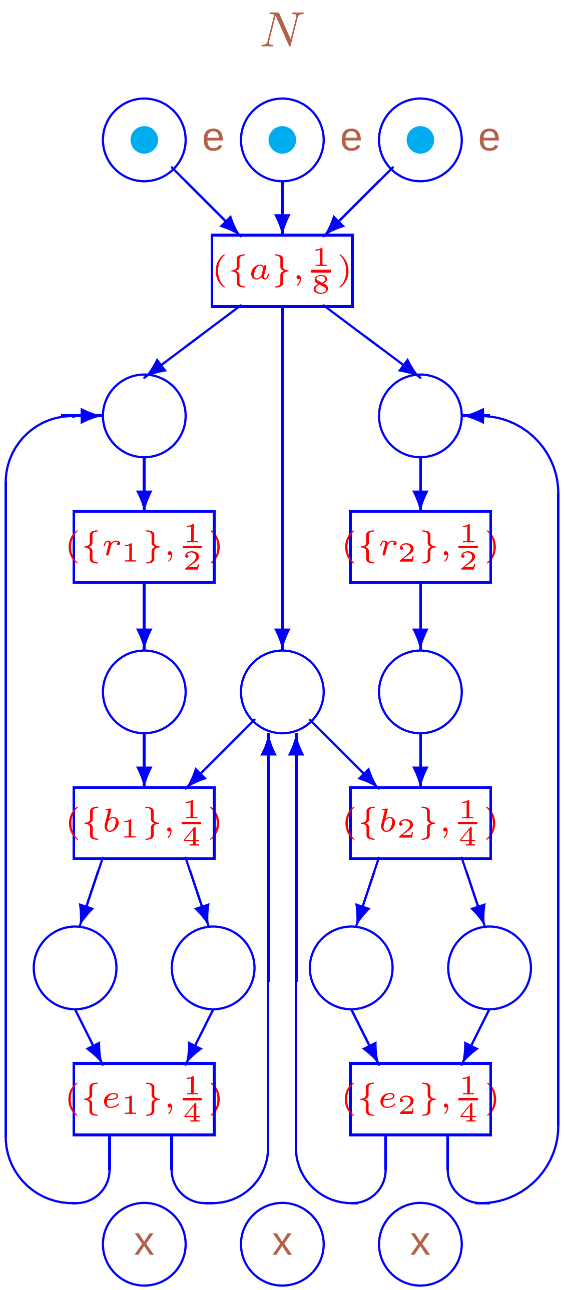
  The steady state probability that the memory is used, the *shared memory utilization*, is $1 - \frac{124}{209} = \frac{85}{209}$.
- The common memory request of the first processor $(\{r_1\}, \frac{1}{2})$ is possible from the states s_2, s_4, s_7 only.

  The request probability in each of the states is a sum of execution probabilities for all multisets of activities containing $(\{r_1\}, \frac{1}{2})$.

  The *steady state probability of the shared memory request from the first processor* is
  $\psi_2^* \sum_{\{\Gamma|(\{r_1\},\frac{1}{2})\in\Gamma\}} PT^*(\Gamma, s_2) + \psi_4^* \sum_{\{\Gamma|(\{r_1\},\frac{1}{2})\in\Gamma\}} PT^*(\Gamma, s_4) + \psi_7^* \sum_{\{\Gamma|(\{r_1\},\frac{1}{2})\in\Gamma\}} PT^*(\Gamma, s_7) = \frac{3}{209} \cdot \left(\frac{1}{3} + \frac{1}{3}\right) + \frac{75}{418} \cdot \left(\frac{3}{5} + \frac{1}{5}\right) + \frac{15}{418} \cdot \left(\frac{3}{5} + \frac{1}{5}\right) = \frac{38}{209}$.



The marked dts-boxes of two processors and shared memory



The marked dts-box of the shared memory system

Overview and open questions

The results obtained

- A discrete time stochastic extension $dtsPBC$ of finite PBC enriched with iteration.
- The step operational semantics based on labeled probabilistic transition systems.
- The denotational semantics in terms of a subclass of LDTSPNs.
- A case study of performance analysis: the shared memory system.

Further research

- Constructing the stochastic equivalences which abstract from empty loops.
- Applying the equivalences to reduction of transition systems.
- Searching for the weakest equivalence that preserves stationary behaviour.
- Introducing the immediate activities with zero delay.
- Extending the syntax with recursion operator.

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The slides can be downloaded from Internet:

`http://www.iis.nsk.su/persons/itar/csp09sld.pdf`

Thank you for your attention!