Investigating Equivalence Notions for Time Petri Nets *

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Abstract

In this paper, in the framework of time Petri nets with fixed time delays and silent transitions we propose a number of timed, untimed and region equivalence notions in both trace and bisimulation semantics. We also consider τ-variants of all these equivalences which abstract of silent actions. Interrelations of all the equivalences are examined for general class of time nets as well as for their subclass of untimed nets with instantaneous transition firings. Preservation of the equivalence relations by new operation of *time SM-refinement* is investigated to have information which relations may be used for top-down design.

Key words & phrases: time and untime Petri nets, silent transitions, timed, untimed and region equivalences, trace and bisimulation semantics.

1 Introduction

An important ingredient of every theory of concurrency is a notion of equivalence between systems. Typically, equivalences are used in the setting of specification and verification both to compare two distinct systems and to reduce the structure of a system. Over the past several years, a variety of equivalences — most notably, perphars, trace and bisimulation ones — have been promoted, and the relationship between them has been quite well-understood (see, for example [7]).

Those equivalences were considered for formal system models without time delays and were not time-sensible. Recently, a growing interest can be observed in modelling real-time systems which imply a need of a representation of the lapse of time. Several formal methods for specifying and reasoning about such systems have been proposed in the last years. Whereas, the incorparation of real time into equivalence notions is less advanced. There are a few papers (see, for example, [2, 4]) where decidability questions of timed equivalences are investigated. In these studies, real-time systems are represented by parallel timer processes or timed automata, containing fictitious time measuring elements called clocks. However, concurrency can not be modelled directly by such timed states graphs. On the other hand, to model real-time systems over dense time domain, time nets were considered in [8, 5]. A time net proceeds in one of two ways: by firing transitions or letting a certain amount of real time pass.

The following basic equivalences were considered in the literature.

- *Timed equivalences* (respect time delays in nets functioning): trace ($\equiv_t$) [2], bisimulation $\underline{\leftrightarrow}_t$ [4, 2].
- *Untimed equivalences* (do not respect time delays in nets functioning): trace ($\equiv_u$) [2], bisimulation $\underline{\leftrightarrow}_u$ [2].
- *Timed τ-equivalences* (respect time delays in nets functioning and abstract of silent actions): bisimulation ($\underline{\leftrightarrow}_t^\tau$) [4].
- *Isomorphism* (coincidence of time nets w.r.t. renaming of places and transitions): ($\simeq$) [8].

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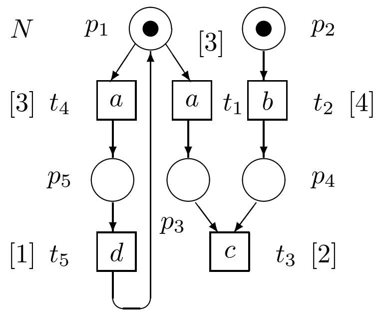

Figure 1: An example of time net

Our main point here is to introduce timed, untimed and region equivalences in the trace and bisimulation cases and establish the interrelations of these equivalences in the framework of time nets with silent transitions. We treat also τ-variants of these notions which take into consideration "invisible" nature of the silent action τ.

We introduce the following new relations: time τ-trace equivalence ($\equiv_t^\tau$), untime τ-trace equivalence ($\equiv_u^\tau$) and untime τ-bisimulation equivalence ($\underline{\leftrightarrow}_u^\tau$) and

We consider also region equivalences which partition the states of a TPN into the so-called "regions" [1], so it makes easier their check than that of timed ones. These are: trace region equivalence ($\equiv_r$), bisimulation region equivalence ($\underline{\leftrightarrow}_r$), τ-trace region equivalence ($\equiv_r^\tau$) and τ-bisimulation region equivalence ($\underline{\leftrightarrow}_r^\tau$).

We compare all these notions on general and sequential time nets to clarify their nature and simplify check of them.

We treat preservation of the equivalence relations by new operation of time SM-refinement to find which of them may be used for top-down design.

2 Basic definitions

Let $Act = \{a, b, c, \dots\}$ be a set of *action names* and τ be the special silent or hidden action which is considered to be invisible for extermal observer. We denote $Act_\tau = Act \cup \{\tau\}$. Let $\mathbf{R}^+$ be a set of *nonnegative real numbers.*

Definition 2.1 *A* time net *is a 6-tuple* $N = \langle P_N, T_N, F_N, l_N, M_N, \Upsilon_N \rangle$, *where:*

- $\langle P_N, T_N, F_N, l_N, M_N \rangle$ *is a safe net without loops (i.e.* $\forall t \in T_N\ ^\bullet t \cap t^\bullet = \emptyset$*) with labelling over* Act_τ*;*
- $\Upsilon_N : T_N \to \mathbf{N}$ *is the* initial time delay *function (satisfying* $\forall t \in T_N\ (l_N(t) = \tau \ \Rightarrow\ \Upsilon_N(t) = 0)$*).*

Example 2.1 *In Figure 1 a graphical representation of a time net is presented. The initial transition delays are depicted by numbers in brackets near corresponding transitions.*

We introduce the following notations. For $x \in T_N \cup P_N$, $^\bullet x = \{y \mid (y, x) \in F_N\}$ and $x^\bullet = \{x \mid (x, y) \in F_N\}$ denote the *preset* and *postset* of x, respectively. We shall need to refer to the set of places without ingoing arcs or without outgoing arcs. Let $^\circ N = \{p \in P_N \mid {}^\bullet p = \emptyset\}$ and $N^\circ = \{p \in P_N \mid p^\bullet = \emptyset\}$.

Given time nets $N = \langle P_N, T_N, F_N, l_N, M_N, \Upsilon_N \rangle$ and $N' = \langle P_{N'}, T_{N'}, F_{N'}, l_{N'}, M_{N'}, \Upsilon_{N'} \rangle$. A mapping $\beta : N \to N'$ is an *isomorphism* between N and N', denoted by $\beta : N \simeq N'$, if:

1. β is a bijection s.t. $\beta(P_N) = P_{N'}$ and $\beta(T_N) = T_{N'}$;
2. $\forall p \in P_N\ \forall t \in T_N\ (p, t) \in F_N \ \Leftrightarrow\ (\beta(p), \beta(t)) \in F_{N'}$ and $(t, p) \in F_N \ \Leftrightarrow\ (\beta(t), \beta(p)) \in F_{N'}$;
3. $\forall t \in T_N\ l_N(t) = l_{N'}(\beta(t))$;
4. $\forall p \in P_N\ p \in M_N \ \Leftrightarrow\ \beta(p) \in M_{N'}$;
5. $\forall t \in T_N\ \Upsilon_N(t) = \Upsilon_{N'}(\beta(t))$.

Two time nets N and N' are *isomorphic*, denoted by $N \simeq N'$, if $\exists \beta : N \simeq N'$.

A *state* of a time net N is a pair $Q = (M, \Upsilon)$, where:

1. $M \in 2^{P_N}$ is a marking of N;

2. $\Upsilon : T_N \to \mathbf{R}^+$ is a *time delay* function.

The *initial* state of a time net N is a pair $Q_N = (M_N, \Upsilon_N)$.

A time net can proceed by firing of a transition or by passing an amount of time (delay).

Let $Q = (M, \Upsilon)$ be a state of a time net N.

A transition $t \in T_N$ is *firable* in Q, if ${}^\bullet t \subseteq M$ and $\Upsilon(t) = 0$. If t is fireable in Q, firing it yields a new state $\widetilde{Q} = (\widetilde{M}, \widetilde{\Upsilon})$, denoted by $Q \stackrel{t}{\to} \widetilde{Q}$, where $\widetilde{Q}$ is defined as follows:

1. $\widetilde{M} = (M \setminus {}^\bullet t) \cup t^\bullet$;
2. $\forall u \in T_N\ \widetilde{\Upsilon}(u) = \begin{cases} \Upsilon_N(u), & ({}^\bullet u \subseteq \widetilde{M}) \wedge ({}^\bullet u \not\subseteq M); \\ \Upsilon(u), & \text{otherwise.} \end{cases}$

We write $Q \stackrel{a}{\to} \widetilde{Q}$, if $\exists t\ Q \stackrel{t}{\to} \widetilde{Q}$ and $l_N(t) = a$.

A time $\delta \in \mathbf{R}^+$ *can pass* in Q, if $\forall u \in T_N$ s.t. ${}^\bullet u \subseteq M$ it holds: $0 < \delta \le \Upsilon(u)$. If δ can pass in Q, passing it yields a new state $\widetilde{Q} = (\widetilde{M}, \widetilde{\Upsilon})$, denoted by $Q \stackrel{\delta}{\to} \widetilde{Q}$, where $\widetilde{Q}$ is defined as follows:

1. $\widetilde{M} = M$;
2. $\forall u \in T_N\ \widetilde{\Upsilon}(u) = \Upsilon(u) \ominus \delta$, where $\forall x, y\ x \ominus y = \max\{0, x - y\}$.

We write $Q \stackrel{\delta}{\to} \widetilde{Q}$, if $\exists \delta_1, \delta_2, Q_1\ Q \stackrel{\delta_1}{\to} Q_1 \stackrel{\delta_2}{\to} \widetilde{Q}$ and $\delta_1 + \delta_2 = \delta$.

A state Q of a time net N is *reachable*, if $Q = Q_N$ or there exists a reachable state $\hat{Q}$ of N s.t. $\hat{Q} \stackrel{x}{\to} Q$ for some $x \in Act_\tau \cup \mathbf{R}$. $States(N)$ denotes the set of *all reachable states* of N.

A *cycle* of a time net N is a set of transitions $\{t_0, \ldots, t_n\} \subseteq T_N$ s.t. $t_{i-1}^\bullet \cap {}^\bullet t_i \neq \emptyset$ $(1 \le i \le n)$ and $t_0 = t_n$. From now on we consider only time nets satisfying the *progress condition*, stating that $\sum_{i=0}^{n} \Upsilon_N(t_i) > 0$ for any cycle $\{t_0, \ldots, t_n\}$ of N.

3 Equivalences

3.1 Timed equivalences

3.1.1 Timed trace equivalences

Definition 3.1 *A* time trace *of a time net N is a sequence $x_1 \cdots x_n \in (Act_\tau \cup \mathbf{R}^+)^*$ s.t. $Q_N \stackrel{x_1}{\to} Q_1 \stackrel{x_2}{\to} \ldots \stackrel{x_n}{\to} Q_n$. We denote a set of* all time traces *of a time net N by $TimeTraces(N)$. Two time nets N and N' are* timed trace equivalent*, denoted by $N \equiv_t N'$, if $TimeTraces(N) = TimeTraces(N')$.*

3.1.2 Timed bisimulation equivalences

Definition 3.2 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq States(N) \times States(N')$ is a* timed bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}_t N'$, if:*

1. *$(Q_N, Q_{N'}) \in \mathcal{R}$.*
2. *$(Q, Q') \in \mathcal{R},\ Q \stackrel{x}{\to} \widetilde{Q}\ (x \in Act_\tau \cup \mathbf{R}^+)\ \Rightarrow\ \exists \widetilde{Q}' :\ Q' \stackrel{x}{\to} \widetilde{Q}',\ (\widetilde{Q}, \widetilde{Q}') \in \mathcal{R}$.*
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are timed bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}_t N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}_t N'$.*

3.2 Untimed equivalences

We shall use the following notations.

We write $Q \stackrel{t}{\mapsto} \widetilde{Q}$, if $\exists \delta_1, \delta_2, Q_1, Q_2\ Q \stackrel{\delta_1}{\to} Q_1 \stackrel{t}{\to} Q_2 \stackrel{\delta_2}{\to} \widetilde{Q}$.

We write $Q \stackrel{a}{\mapsto} \widetilde{Q}$, if $\exists t\ Q \stackrel{t}{\mapsto} \widetilde{Q}$ and $l_N(t) = a$.

3.2.1 Untimed trace equivalences

Definition 3.3 *An* untime trace *of a time net N is a sequence $a_1 \cdots a_n \in Act_\tau^*$ s.t. $Q_N \stackrel{a_1}{\mapsto} Q_1 \stackrel{a_2}{\mapsto} \ldots \stackrel{a_n}{\mapsto} Q_n$. We denote a set of* all untime traces *of a time net N by $UntimeTraces(N)$. Two time nets N and N' are* untimed trace equivalent*, denoted by $N \equiv_u N'$, if $UnimeTraces(N) = UntimeTraces(N')$.*

3.2.2 Untimed bisimulation equivalences

Definition 3.4 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq States(N) \times States(N')$ is an* untimed bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}_u N'$, if:*

1. $(Q_N, Q_{N'}) \in \mathcal{R}$.
2. $(Q, Q') \in \mathcal{R},\ Q \overset{a}{\mapsto} \widetilde{Q}\ (a \in Act_\tau)\ \Rightarrow\ \exists \widetilde{Q}' :\ Q' \overset{a}{\mapsto} \widetilde{Q}',\ (\widetilde{Q}, \widetilde{Q}') \in \mathcal{R}$.
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are untimed bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}_u N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}_u N'$.*

3.3 Region equivalences

For $\delta \in \mathbf{R}^+$, let $\lceil \delta \rceil$ denote its integral part, and $\{\delta\}$ denote its fractional part.

Let $Q_1 = (M_1, \Upsilon_1)$ and $Q_2 = (M_2, \Upsilon_2)$ be states of a time net N. Two time delay functions Υ_1 and Υ_2 are *region equivalent*, denoted by $\Upsilon_1 =_{reg} \Upsilon_2$, if:

1. $\forall t \in T_N\ \lceil \Upsilon_1(t) \rceil = \lceil \Upsilon_2(t) \rceil$;
2. (a) $\forall t \in T_N\ \{\Upsilon_1(t)\} = 0\ \Leftrightarrow\ \{\Upsilon_2(t)\} = 0$;

   (b) $\forall t, u \in T_N\ \{\Upsilon_1(t)\} \le \{\Upsilon_1(u)\}\ \Leftrightarrow\ \{\Upsilon_2(t)\} \le \{\Upsilon_2(u)\}$.

Two states Q_1 and Q_2 are *region equivalent*, denoted by $Q_1 =_{reg} Q_2$, if $M_1 = M_2$ and $\Upsilon_1 =_{reg} \Upsilon_2$.

A *time region* of a state Q is defined as: $[Q] = [Q]_{=_{reg}} = \{R \mid R =_{reg} Q\}$. A set of *all reachable region states* of a time net N is defined as follows: $RegStates(N) = States(N)/=_{reg} = \{[Q] \mid Q \in States(N)\}$.

Example 3.1 *Let us find the time region of a state $Q = (M, \Upsilon)$ of a time net N with seven transitions. If $\Upsilon(t_i) = \delta_i$ $(1 \le i \le 7)$ and $\delta_1 = 0.1,\ \delta_2 = 5,\ \delta_3 = 1.33,\ \delta_4 = 7.42,\ \delta_5 = 7.999,\ \delta_6 = 5,\ \delta_7 = 0$, then the corresponding time region $[Q]$ has transition delays defined by the following inequations:*

1. $0 = \delta_7 < \delta_1 < 1 < \delta_3 < 2 < 5 = \delta_2 = \delta_6 < 7 < \delta_4 < \delta_5 < 8$;
2. $0 = \{\delta_2\} = \{\delta_6\} = \{\delta_7\} < \{\delta_1\} < \{\delta_3\} < \{\delta_4\} < \{\delta_5\}$.

Let $Q = (M, \Upsilon)$ be a state of a time net N and $Q' = (M', \Upsilon')$ be a state of a time net N'. We introduce the following notations.

$$\zeta(Q) = \begin{cases} 1/2, & \forall t \in T_N\ \Upsilon(t) \in \mathbf{N}; \\ \min\{\{\Upsilon(t)\} \mid t \in T_N\}, & \text{otherwise.} \end{cases}$$

$$\zeta(Q, Q') = \min\{\zeta(Q), \zeta(Q')\}$$

A state Q is *stable*, if $\exists \delta > 0\ \exists \widetilde{Q}\ Q \overset{\delta}{\to} \widetilde{Q}$. In such a case, we denote the state $\widetilde{Q}$ by $Q(\delta)$.

We shall use the following notations.

We write $[Q] \overset{t}{\to} [\widetilde{Q}]$, if $Q \overset{t}{\to} \widetilde{Q}$.

We write $[Q] \overset{a}{\to} [\widetilde{Q}]$, if $\exists t\ [Q] \overset{t}{\to} [\widetilde{Q}]$ and $l_N(t) = a$.

We write $[Q] \overset{\surd}{\to} [\widetilde{Q}]$, if $Q \overset{\zeta(Q)}{\to} \widetilde{Q}$.

3.3.1 Region trace equivalence

Definition 3.5 *A* region trace *of a time net N is a sequence $x_1 \cdots x_n \in (Act_\tau \cup \{\surd\})^*$ s.t. $[Q_N] \overset{x_1}{\to} [Q_1] \overset{x_2}{\to} \ldots \overset{x_n}{\to} [Q_n]$. We denote a set of* all region traces *of a time net N by $RegTraces(N)$. Two time nets N and N' are* region trace equivalent*, denoted by $N \equiv_r N'$, if $RegTraces(N) = RegTraces(N')$.*

3.3.2 Region bisimulation equivalence

Definition 3.6 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq RegStates(N) \times RegStates(N')$ is a* region bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}_r N'$, if:*

1. $([Q_N], [Q_{N'}]) \in \mathcal{R}$.
2. $([Q], [Q']) \in \mathcal{R}$,
   (a) $[Q] \stackrel{a}{\to} [\widetilde{Q}]$ $(a \in Act_\tau)$ $\Rightarrow$ $\exists [\widetilde{Q}'] :$ $[Q'] \stackrel{a}{\to} [\widetilde{Q}']$, $([\widetilde{Q}], [\widetilde{Q}']) \in \mathcal{R}$;
   (b) *if Q is stable and $\zeta = \zeta(Q, Q')$ then $([Q(\zeta)], [Q'(\zeta)]) \in \mathcal{R}$.*
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are region bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}_r N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}_r N'$.*

4 τ-equivalences

4.1 Timed τ-equivalences

We shall use the following notations.

We write $Q \Rightarrow \widetilde{Q}$, if $\exists Q_i$ $(1 \le i \le n)$ $Q \stackrel{\tau}{\to} Q_1 \stackrel{\tau}{\to} \cdots \stackrel{\tau}{\to} Q_n = \widetilde{Q}$.
We write $Q \stackrel{t}{\Rightarrow} \widetilde{Q}$, if $\exists Q_1, Q_2$ $Q \Rightarrow Q_1 \stackrel{t}{\to} Q_2 \Rightarrow \widetilde{Q}$.
We write $Q \stackrel{a}{\Rightarrow} \widetilde{Q}$, if $\exists t$ $Q \stackrel{t}{\Rightarrow} \widetilde{Q}$ and $l_N(t) = a$.
We write $Q \stackrel{\delta}{\Rightarrow} \widetilde{Q}$, if $\exists Q_1$ $Q \Rightarrow Q_1 \stackrel{\delta}{\to} \widetilde{Q}$.
We write $Q \stackrel{\delta}{\Rightarrow} \widetilde{Q}$, if $\exists \delta_1, \delta_2, Q_1$ $Q \stackrel{\delta_1}{\Rightarrow} Q_1 \stackrel{\delta_2}{\Rightarrow} \widetilde{Q}$ and $\delta_1 + \delta_2 = \delta$.
Let us introduce an *empty symbol* '_' which denotes "nothing" or empty alternative.

4.1.1 Timed τ-trace equivalences

Definition 4.1 *A* visible time trace *of a time net N is a sequence $y_1 \cdots y_n \in (Act \cup \mathbf{R}^+)^*$ s.t. $Q_N \stackrel{y_1}{\Rightarrow} Q_1 \stackrel{y_2}{\Rightarrow} \ldots \stackrel{y_n}{\Rightarrow} Q_n$. We denote a set of* all visible time traces *of a time net N by $VisTimeTraces(N)$. Two time nets N and N' are* timed τ-trace equivalent*, denoted by $N \equiv_t^\tau N'$, if $VisTimeTraces(N) = VisTimeTraces(N')$.*

4.1.2 Timed τ-bisimulation equivalences

Definition 4.2 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq States(N) \times States(N')$ is a* timed τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}_t^\tau N'$, if:*

1. $(Q_N, Q_{N'}) \in \mathcal{R}$.
2. $(Q, Q') \in \mathcal{R}$, $Q \stackrel{y}{\Rightarrow} \widetilde{Q}$ $(y \in Act \cup \mathbf{R}^+ \cup \{\_\})$ $\Rightarrow$ $\exists \widetilde{Q}' :$ $Q' \stackrel{y}{\Rightarrow} \widetilde{Q}'$, $(\widetilde{Q}, \widetilde{Q}') \in \mathcal{R}$.
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are timed τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}_t^\tau N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}_t^\tau N'$.*

4.2 Untimed τ-equivalences

4.2.1 Untimed τ-trace equivalences

We shall use the following notations.

We write $Q \models\!\!\!\Rightarrow \widetilde{Q}$, if $\exists \delta$ $Q \stackrel{\delta}{\models\!\!\!\Rightarrow} \widetilde{Q}$.
We write $Q \stackrel{t}{\models\!\!\!\Rightarrow} \widetilde{Q}$, if $\exists Q_1, Q_2$ $Q \models\!\!\!\Rightarrow Q_1 \stackrel{t}{\Rightarrow} Q_2 \models\!\!\!\Rightarrow \widetilde{Q}$.
We write $Q \stackrel{a}{\models\!\!\!\Rightarrow} \widetilde{Q}$, if $\exists t$ $Q \stackrel{t}{\models\!\!\!\Rightarrow} \widetilde{Q}$ and $l_N(t) = a$.

Definition 4.3 *A* visible untime trace *of a time net N is a sequence $b_1 \cdots b_n \in Act^*$ s.t. $Q_N \stackrel{b_1}{\models\!\!\!\Rightarrow} Q_1 \stackrel{b_2}{\models\!\!\!\Rightarrow} \ldots \stackrel{b_n}{\models\!\!\!\Rightarrow} Q_n$. We denote a set of* all visible untime traces *of a time net N by $VisUntimeTraces(N)$. Two time nets N and N' are* untimed τ-trace equivalent*, denoted by $N \equiv_u^\tau N'$, if $VisUntimeTraces(N) = VisUnimeTraces(N')$.*

4.2.2 Untimed τ-bisimulation equivalences

Definition 4.4 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq States(N) \times States(N')$ is an* untimed τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{u} N'$, if:*

1. $(Q_N, Q_{N'}) \in \mathcal{R}$.
2. $(Q, Q') \in \mathcal{R},\ Q \overset{b}{\models\!\!\Rightarrow} \widetilde{Q}\ (b \in Act \cup \{\_\})\ \Rightarrow\ \exists \widetilde{Q}' :\ Q' \overset{b}{\models\!\!\Rightarrow} \widetilde{Q}',\ (\widetilde{Q}, \widetilde{Q}') \in \mathcal{R}$.
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are untimed τ-bisimulation equivalent, *denoted by $N \underline{\leftrightarrow}^{\tau}_{u} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{u} N'$.*

4.3 Region τ-equivalences

Let Q be a state of a time net N s.t. $\exists \delta > 0\ \exists \widetilde{Q}\ Q \overset{\delta}{\Rightarrow} \widetilde{Q}$. In such a case we denote $Q(\!|\delta|\!) = \{\widetilde{Q} \mid Q \overset{\delta}{\Rightarrow} \widetilde{Q}\}$.

We shall use the following notations.

We write $[Q] \Rightarrow [\widetilde{Q}]$, if $Q \Rightarrow \widetilde{Q}$.

We write $[Q] \overset{t}{\Rightarrow} [\widetilde{Q}]$, if $Q \overset{t}{\Rightarrow} \widetilde{Q}$.

We write $[Q] \overset{a}{\Rightarrow} [\widetilde{Q}]$, if $\exists t\ [Q] \overset{t}{\Rightarrow} [\widetilde{Q}]$ and $l_N(t) = a$.

We write $[Q] \overset{\surd}{\Rightarrow} [\widetilde{Q}]$, if $Q \overset{\zeta(Q)}{\Rightarrow} \widetilde{Q}$.

4.3.1 Region τ-trace equivalence

Definition 4.5 *A* visible region trace *of a time net N is a sequence $y_1 \cdots y_n \in (Act \cup \{\surd\})^*$ s.t. $[Q_N] \overset{y_1}{\Rightarrow} [Q_1] \overset{y_2}{\Rightarrow} \dots \overset{y_n}{\Rightarrow} [Q_n]$. We denote a set of* all visible region traces *of a time net N by $VisRegTraces(N)$. Two time nets N and N' are* region τ-trace equivalent, *denoted by $N \equiv^{\tau}_{r} N'$, if $VisRegTraces(N) = VisRegTraces(N')$.*

4.3.2 Region τ-bisimulation equivalence

Definition 4.6 *Let N and N' be some time nets. A relation $\mathcal{R} \subseteq RegStates(N) \times RegStates(N')$ is a* region τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{r} N'$, if:*

1. $([Q_N], [Q_{N'}]) \in \mathcal{R}$.
2. $([Q], [Q']) \in \mathcal{R}$,
   (a) $[Q] \overset{b}{\rightarrow} [\widetilde{Q}]\ (b \in Act \cup \{\_\})\ \Rightarrow\ \exists [\widetilde{Q}'] :\ [Q'] \overset{b}{\Rightarrow} [\widetilde{Q}'],\ ([\widetilde{Q}], [\widetilde{Q}']) \in \mathcal{R}$;
   (b) *if Q is stable and $\zeta = \zeta(Q, Q')$ then $\exists \widetilde{Q}' \in Q'(\!|\zeta|\!) :\ ([Q(\zeta)], [\widetilde{Q}']) \in \mathcal{R}$.*
3. *As item 2, but the roles of N and N' are reversed.*

Two time nets N and N' are region τ-bisimulation equivalent, *denoted by $N \underline{\leftrightarrow}^{\tau}_{r} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{r} N'$.*

5 Interrelations of the equivalences and τ-equivalences

In this section we show the coincidence of the region equivalence notions with timed ones which provides a tool to reduce the number of states of a TPN implying the simplification of the timed equivalences check, and establish interrelations of all the equivalence notions.

A mapping $\kappa : \mathbf{R}^+ \to \mathbf{R}^+$ is *uniform*, if:

1. $\kappa(0) = 0$;
2. $\forall n \in \mathbf{N}\ \kappa(x + n) = \kappa(x) + n$.

Let $Q = (M, \Upsilon)$ be state of a time net N and $Q' = (M, \Upsilon)$ be state of a time net N'. We write $\kappa(Q) = (M, \kappa \circ \Upsilon)$.

Lemma 5.1 *[4]*

1. *Let $Q_1 = (M_1, \Upsilon_1), Q_2 = (M_2, \Upsilon_2)$ be states of a time net N. Then $Q_1 =_{reg} Q_2$ iff there exists uniform mapping κ s.t. $\kappa(Q_1) = (Q_2)$.*

2. *If κ is a uniform mapping then $\forall \delta \in \mathbf{R}^+$ the mapping κ_δ, defined as $\forall x \in \mathbf{R}^+\ \kappa_\delta(x) = \kappa(x+\delta) - \kappa(\delta)$, is also uniform.*

Proposition 5.1 *Let N be some time net, $Q_1, Q_2 \in States(N)$ and κ be some uniform mapping s.t. $\kappa(Q_1) = Q_2$. Then:*

1. *$Q_1 \xrightarrow{t} \widetilde{Q}_1$ implies $Q_2 \xrightarrow{t} \kappa(\widetilde{Q}_1)$ for some $t \in T_N$;*
2. *$Q_1 \xrightarrow{\delta} \widetilde{Q}_1$ implies $Q_2 \xrightarrow{\kappa(\delta)} \kappa_\delta(\widetilde{Q}_1)$ for some $\delta \in \mathbf{R}^+$.*

Proof. Let $Q_1 = (M, \Upsilon)$. Then $Q_2 = (M, \kappa \circ \Upsilon)$.

1. Let $Q_1 \xrightarrow{t} \widetilde{Q}_1$, where $\widetilde{Q}_1 = (\widetilde{M}, \widetilde{\Upsilon})$. Then $\Upsilon(t) = 0$. Since $\kappa(\Upsilon(t)) = \kappa(0) = 0$, we have $Q_2 \xrightarrow{t} \widetilde{Q}_2$, where $\widetilde{Q}_2 = (\widetilde{M}, \widetilde{\Upsilon}_2)$ and

$$\forall u \in T_N\ \widetilde{\Upsilon}_2(u) = \begin{cases} \Upsilon_N(u), & ({}^\bullet u \subseteq \widetilde{M}) \wedge ({}^\bullet u \not\subseteq M); \\ \kappa(\Upsilon(u)), & \text{otherwise.} \end{cases}$$

   Since $\forall u \in T_N\ \Upsilon_N(u) \in \mathbf{N}$, then $\Upsilon_N(u) = \kappa(\Upsilon_N(u))$. Hence $\widetilde{\Upsilon}_2 = \kappa \circ \widetilde{\Upsilon}$ and $\widetilde{Q}_2 = \kappa(\widetilde{Q}_1)$.

2. Let $Q_1 \xrightarrow{\delta} \widetilde{Q}_1$, where $\widetilde{Q}_1 = (\widetilde{M}, \widetilde{\Upsilon})$ and $\widetilde{\Upsilon} = \Upsilon \ominus \delta$. Then $\forall u \in T_N$ s.t. ${}^\bullet u \subseteq M$ it holds: $0 < \delta \leq \Upsilon(u)$. Since κ is a strict monotonous mapping, then $0 = \kappa(0) < \kappa(\delta) \leq \kappa(\Upsilon(u))$, and we have $Q_2 \xrightarrow{\kappa(\delta)} \widetilde{Q}_2$, where $\widetilde{Q}_2 = (\widetilde{M}, \widetilde{\Upsilon}_2)$ and $\widetilde{\Upsilon}_2 = \kappa \circ \Upsilon \ominus \kappa(\delta)$.

   Let $u \in T_N$. We have the following two cases.

   (a) $\Upsilon(u) \geq \delta$.
   Then $\widetilde{\Upsilon}(u) = \Upsilon(u) \ominus \delta = \Upsilon(u) - \delta$.
   Since κ is strict monotonous, we have $\kappa(\Upsilon(u)) \geq \kappa(\delta)$ and $\kappa(\Upsilon(u)) \ominus \kappa(\delta) = \kappa(\Upsilon(u)) - \kappa(\delta) = \kappa(\widetilde{\Upsilon}(u) + \delta) - \kappa(\delta) = \kappa_\delta(\widetilde{\Upsilon}(u))$.

   (b) $\Upsilon(u) < \delta$.
   Then $\widetilde{\Upsilon}(u) = \Upsilon(u) \ominus \delta = 0$.
   Since κ is strict monotonous, we have $\kappa(\Upsilon(u)) < \kappa(\delta)$ and $\kappa(\Upsilon(u)) \ominus \kappa(\delta) = 0 = \kappa_\delta(0) = \kappa_\delta(\widetilde{\Upsilon}(u))$.

   Hence $\widetilde{\Upsilon}_2 = \kappa_\delta \circ \widetilde{\Upsilon}$ and $\widetilde{Q}_2 = \kappa_\delta(\widetilde{Q}_1)$. □

Proposition 5.2 *Let N and N' be some time nets, $Q_1, Q_2 \in States(N)$, $Q_1', Q_2' \in States(N')$. Then $Q_1 =_{reg} Q_2$ and $Q_1' =_{reg} Q_2'$ implies: $Q_1 \equiv_t Q_1' \ \Leftrightarrow\ Q_2 \equiv_t Q_2'$.*

Proof. As for timed bisimulation equivalence in Proposition 5.3, but simpler. □

Let for some time nets N and N' $Q \in States(N)$, $Q' \in States(N')$. We write $Q \underline{\leftrightarrow}_t Q'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}_t N'$ s.t. $(Q, Q') \in \mathcal{R}$.

Proposition 5.3 *Let N and N' be some time nets, $Q_1, Q_2 \in States(N)$, $Q_1', Q_2' \in States(N')$. Then $Q_1 =_{reg} Q_2$ and $Q_1' =_{reg} Q_2'$ implies: $Q_1 \underline{\leftrightarrow}_t Q_1' \ \Leftrightarrow\ Q_2 \underline{\leftrightarrow}_t Q_2'$.*

Proof. ($\Leftarrow$) Let us define a relation $\mathcal{R}$ as follows: $\mathcal{R} = \{(Q_1, Q_1') \mid \exists Q_2 \in States(N)\ \exists Q_2' \in States(N')\ (Q_1 =_{reg} Q_2) \wedge (Q_1' =_{reg} Q_2') \wedge (Q_2 \underline{\leftrightarrow}_t Q_2')\}$. All we should do is to prove that $\mathcal{R}$ has the transfer property. Since the property is symmetrical, it is enough to check only simulation of N by N'.

Let $(Q_1, Q_1') \in \mathcal{R}$. Then $\exists (Q_2, Q_2')$ s.t. $Q_1 =_{reg} Q_2$, $Q_1' =_{reg} Q_2'$ and $Q_2 \underline{\leftrightarrow}_t Q_2'$. Hence for some uniform mappings κ and κ' we have $Q_2 = \kappa(Q_1)$ and $Q_2' = \kappa'(Q_1')$.

1. Let $Q_1 \xrightarrow{a} \widetilde{Q}_1$ ($a \in Act_\tau$). By Proposition 5.1, $Q_2 \xrightarrow{a} \kappa(\widetilde{Q})$.

   Since $Q_2 \underline{\leftrightarrow}_t Q_2'$ then $\exists \widetilde{Q}_2' :\ Q_2' \xrightarrow{a} \widetilde{Q}_2'$ and $\kappa(\widetilde{Q}_1) \underline{\leftrightarrow}_t \widetilde{Q}_2'$. We have $Q' = (\kappa')^{-1}(Q_2')$. Since the mapping inverse to the uniform one is also uniform, by Proposition 5.1 we have $Q_1' \xrightarrow{a} (\kappa')^{-1}(\widetilde{Q}_1')$ and $\widetilde{Q}_1 \underline{\leftrightarrow}_t (\kappa')^{-1}(\widetilde{Q}_2')$.

   Since $\widetilde{Q}_1 =_{reg} \kappa(\widetilde{Q}_1)$ and $(\kappa')^{-1}(\widetilde{Q}_2') =_{reg} \widetilde{Q}_2'$, we have $(\widetilde{Q}_1, (\kappa')^{-1}(\widetilde{Q}_2')) \in \mathcal{R}$.

2. The case $Q_1 \xrightarrow{\delta} \widetilde{Q}_1$ ($\delta \in \mathbf{R}^+$) is considered analogously.

(⇒) Symmetrically to the previous item. □

Proposition 5.4 *For time nets N and N' $N \equiv_t N' \Leftrightarrow N \equiv_r N'$.*

Proof. As for timed bisimulation equivalence in Proposition 5.5, but simpler. □

Proposition 5.5 *For time nets N and N' $N \underline{\leftrightarrow}_t N' \Leftrightarrow N \underline{\leftrightarrow}_r N'$.*

Proof. (⇐) Let $\mathcal{R} : N \underline{\leftrightarrow}_r N'$. We define a relation $\mathcal{S}$ as follows: $\mathcal{S} = \{(Q, Q') \mid ([Q], [Q']) \in \mathcal{R}\}$. Let us prove $\mathcal{S} : N \underline{\leftrightarrow}_t N'$.

1. $(Q_N, Q_{N'}) \in \mathcal{S}$, since $([Q_N], [Q_{N'}]) \in \mathcal{R}$.
2. Let $(Q, Q') \in \mathcal{S}$. All transition firings of N are imitated by N' due to the Proposition 5.1. Let us consider time passings. We have $\forall([Q], [Q']) \in \mathcal{R}$ if Q is stable and $\zeta = \zeta(Q, Q')$ then $([Q(\zeta)], [Q'(\zeta)]) \in \mathcal{R}$. Hence $\forall (Q, Q') \in \mathcal{S}$ if Q is stable then $(Q(\zeta), Q'(\zeta)) \in \mathcal{S}$.

   Let us prove $\forall \delta > 0$ $Q \stackrel{\delta}{\to} \widetilde{Q}$ implies $\exists \widetilde{Q}' : Q' \stackrel{\delta}{\to} \widetilde{Q}'$ and $(\widetilde{Q}, \widetilde{Q}') \in \mathcal{S}$. For this, we consider a sequence of pairs (Q_i, Q'_i) $(i \geq 0)$ s.t.:

   (a) $Q_0 = Q,\ Q'_0 = Q'$;

   (b) $Q_{i+1} = Q_i(\zeta_i),\ Q'_{i+1} = Q'_i(\zeta_i)$, where $\zeta_i = \zeta(Q_i, Q'_i)$;

   (c) $(Q_i, Q'_i) \in \mathcal{S}$.

   Then $\exists n \in \mathbf{N}$ $\sum_{i=0}^{n} \zeta_i \geq \delta$, and we have proved.
3. As item 2, but the roles of N and N' are reversed.

(⇒) By definitions. □

Proposition 5.6 *Let N be some time net, $Q_1, Q_2 \in States(N)$ and κ be some uniform mapping s.t. $\kappa(Q_1) = Q_2$. Then:*

1. *$Q_1 \stackrel{t}{\Rightarrow} \widetilde{Q}_1$ implies $Q_2 \stackrel{t}{\Rightarrow} \kappa(\widetilde{Q}_1)$ for some $t \in T_N$;*
2. *$Q_1 \stackrel{\delta}{\Rightarrow} \widetilde{Q}_1$ implies $Q_2 \stackrel{\kappa(\delta)}{\Rightarrow} \kappa_\delta(\widetilde{Q}_1)$ for some $\delta \in \mathbf{R}^+$.*

Proof.

1. The case $Q_1 \stackrel{t}{\Rightarrow} \widetilde{Q}_1$ is considered as in Proposition 5.1, using induction by intermediate states.
2. The case $Q_1 \stackrel{\delta}{\Rightarrow} \widetilde{Q}_1$ is considered as in Proposition 5.1, using induction by intermediate states and the following note: $(\kappa_{\delta_1})_{\delta_2}(x) = \kappa_{\delta_1}(x + \delta_2) - \kappa_{\delta_1}(\delta_2) = \kappa(x + (\delta_1 + \delta_2)) - \kappa(\delta_1) - (\kappa(\delta_1 + \delta_2) - \kappa(\delta_1)) = \kappa(x + (\delta_1 + \delta_2)) - \kappa(\delta_1 + \delta_2) = \kappa_{(\delta_1+\delta_2)}(x)$. □

Proposition 5.7 *Let N and N' be some time nets, $Q_1, Q_2 \in States(N)$, $Q'_1, Q'_2 \in States(N')$. Then $Q_1 =_{reg} Q_2$ and $Q'_1 =_{reg} Q'_2$ implies: $Q_1 \equiv^\tau_t Q'_1 \Leftrightarrow Q_2 \equiv^\tau_t Q'_2$.*

Proof. Analogously to Proposition 5.2. □

Let for some time nets N and N' $Q \in States(N)$, $Q' \in States(N')$. We write $Q \underline{\leftrightarrow}^\tau_t Q'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^\tau_t N'$ s.t. $(Q, Q') \in \mathcal{R}$.

Proposition 5.8 *Let N and N' be some time nets, $Q_1, Q_2 \in States(N)$, $Q'_1, Q'_2 \in States(N')$. Then $Q_1 =_{reg} Q_2$ and $Q'_1 =_{reg} Q'_2$ implies: $Q_1 \underline{\leftrightarrow}^\tau_t Q'_1 \Leftrightarrow Q_2 \underline{\leftrightarrow}^\tau_t Q'_2$.*

Proof. Analogously to Proposition 5.3. □

Proposition 5.9 *For time nets N and N' $N \equiv^\tau_t N' \Leftrightarrow N \equiv^\tau_r N'$.*

Proof. As for timed bisimulation τ-equivalence in Proposition 5.10, but simpler. □

Proposition 5.10 *For time nets N and N' $N \underline{\leftrightarrow}^\tau_t N' \Leftrightarrow N \underline{\leftrightarrow}^\tau_r N'$.*

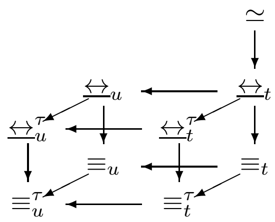

Figure 2: Interrelations of the equivalences and τ-equivalences

Proof. Analogously to the Proposition 5.5, but using Proposition 5.6 instead of 5.1 and $\widetilde{Q}'_i \in Q'_i(\!|\zeta_i|\!)$ instead of $Q'_i(\zeta_i)$ $(1 \le i \le n)$. □

The following theorem establishes the interrelations between the timed equivalence relations (the symbol '_' denotes "nothing").

Theorem 5.1 *Let* $\leftrightarrow, \twoheadleftarrow\!\!\twoheadrightarrow \in \{\equiv, \underline{\leftrightarrow}, \equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star, \star\star \in \{\_, t, u\}$. *For time nets* N *and* N' $N \leftrightarrow_\star N' \;\Rightarrow\; N \twoheadleftarrow\!\!\twoheadrightarrow_{\star\star} N'$ *iff in the graph in Figure 2 there exists a directed path from* $\leftrightarrow_\star$ *to* $\twoheadleftarrow\!\!\twoheadrightarrow_{\star\star}$.

Proof. $(\Leftarrow)$

- The implications $\leftrightarrow_t \to \leftrightarrow_u$, $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \equiv^\tau, \underline{\leftrightarrow}^\tau\}$, are valid, since the result of time abstraction are weaker equivalence notions.
- The implications $\underline{\leftrightarrow}_\star \to \equiv_\star$, $\star \in \{t, u\}$, are valid, since bisimulation equivalences imply trace ones.
- The implications $\underline{\leftrightarrow}^\tau_\star \to \equiv^\tau_\star$, $\star \in \{t, u\}$, are valid, since bisimulation equivalences imply trace ones.
- The implications $\leftrightarrow_t \to \leftrightarrow^\tau_t$, $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$, are valid, since the result of abstraction of silent actions are weaker equivalence notions.
- The implications $\leftrightarrow_u \to \leftrightarrow^\tau_u$, $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$, are valid, since the result of abstraction of silent actions are weaker equivalence notions.
- The implication $\simeq \to \underline{\leftrightarrow}_t$ is obvious.

$(\Rightarrow)$

- In Figure 3(a) $N \underline{\leftrightarrow}_u N'$ but $N \not\equiv_t N'$, since only in the time net N' 1 unit of time can pass before occurrence of action a.
- In Figure 3(b) $N \equiv_t N'$ but $N \underline{\not\leftrightarrow}_u N'$, since only in the time net N' action a can happen so that action b cannot happen afterwards.
- In Figure 3(c) $N \underline{\leftrightarrow}^\tau_t N'$ but $N \not\equiv_u N'$, since only in the time net N' an action τ can happen.
- In Figure 3(d) $N \underline{\leftrightarrow}_t N'$ but $N \not\simeq N'$, since unfireable transitions of time nets N and N' are labelled by different actions (a and b). □

6 The equivalences on untime nets

Definition 6.1 *An* untime net *is a time net* $N = \langle P_N, T_N, F_N, l_N, M_N, \Upsilon_N \rangle$ *s.t.* $\forall t \in T_N \;\; \Upsilon_N(t) = 0$.

For untime nets the coincidence of the timed and untimed equivalences is established, reported in the following proposition.

Proposition 6.1 *Let* $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \equiv^\tau, \underline{\leftrightarrow}^\tau\}$. *For untime nets* N *and* N' $N \leftrightarrow_u N' \;\Leftrightarrow\; N \leftrightarrow_t N'$.

Proof. Obvious, since time cannot pass in untime nets. □

Theorem 6.1 *Let* $\leftrightarrow, \twoheadleftarrow\!\!\twoheadrightarrow \in \{\equiv_u, \underline{\leftrightarrow}_u, \equiv^\tau_u, \underline{\leftrightarrow}^\tau_u, \simeq\}$. *For untime nets* N *and* N' $N \leftrightarrow N' \;\Rightarrow\; N \twoheadleftarrow\!\!\twoheadrightarrow N'$ *iff in the graph in Figure 4 there exists a directed path from* $\leftrightarrow$ *to* $\twoheadleftarrow\!\!\twoheadrightarrow$.

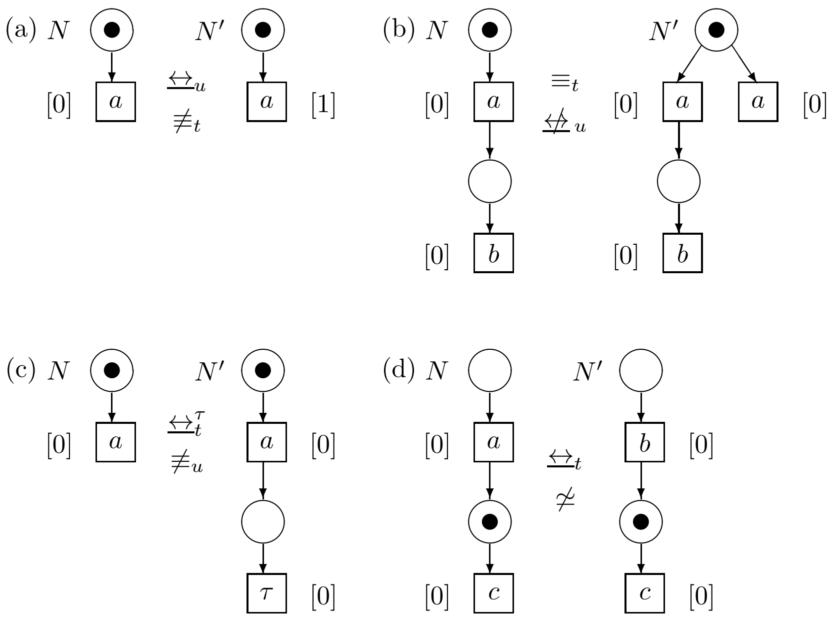

Figure 3: Examples of the equivalences

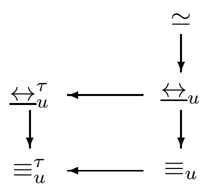

Figure 4: Interrelations of the equivalences and τ-equivalences on untime nets

Proof. ($\Leftarrow$) By Theorem 5.1.

($\Rightarrow$)

- In Figure 3(b) $N \equiv_u N'$ but $N \underline{\not\leftrightarrow}_u N'$.
- In Figure 3(c) $N \underline{\leftrightarrow}_u^\tau N'$ but $N \not\equiv_u N'$.
- In Figure 3(d) $N \underline{\leftrightarrow}_u N'$ but $N \not\simeq N'$. □

7 Preservation of the equivalences by refinements

One of the most important features of an equivalence notion is its stability over refinement of actions. Since we introduce a number equivalences, it is interesting to see whether or not they are resistant to this operation. In our context, this means that if two timed nets are equivalent and we transform them accordingly, whether or not the transformed timed nets will be again equivalent. Incorporating a time notion into the well-known SM-refinement [3], we consider refinement of some transitions of a timed net by a special subclass of time state-machine nets.

Definition 7.1 *A* time SM-net *is a time net* $D = \langle P_D, T_D, F_D, l_D, M_D, \Upsilon_D\rangle$ *s.t.:*

1. $\forall t \in T_D\ |{}^\bullet t| = |t^\bullet| = 1$, *i.e. each transition has exactly one input and one output place;*
2. $\exists p_{in},\ p_{out} \in P_D$ *s.t.* $p_{in} \neq p_{out}$ *and* ${}^\circ D = \{p_{in}\},\ D^\circ = \{p_{out}\}$, *i.e. net D has unique input and unique output place;*
3. $M_D = \{p_{in}\}$, *i.e. at the beginning there is unique token in* p_{in}.
4. *for any two sequences* $(\{p_{in}\}, \Upsilon_D) \overset{t_1}{\mapsto} (M_1, \Upsilon_1) \overset{t_2}{\mapsto} \ldots \overset{t_n}{\mapsto} (M_n, \Upsilon_n) = (\{p_{out}\}, \Upsilon_n)$ *and* $(\{p_{in}\}, \Upsilon_D) \overset{\bar{t}_1}{\mapsto} (\overline{M}_1, \overline{\Upsilon}_1) \overset{\bar{t}_2}{\mapsto} \ldots \overset{\bar{t}_m}{\mapsto} (\overline{M}_m, \overline{\Upsilon}_m) = (\{p_{out}\}, \overline{\Upsilon}_m)$ *we have* $\sum_{i=1}^n \Upsilon_D(t_i) = \sum_{j=1}^m \Upsilon_D(\bar{t}_j) = \Upsilon(D)$.

An action $a \in Act_\tau$ is *conflicting*, if $\exists Q = (M, \Upsilon) \in States(N)\ \exists t, u \in T_N$ s.t. ${}^\bullet t \subseteq M,\ {}^\bullet u \subseteq M,\ {}^\bullet t \cup {}^\bullet u \not\subseteq M,\ \Upsilon(t) = \Upsilon(u) = 0$ and $l_N(t) = a$.

An action $a \in Act_\tau$ is *autoconcurrent*, if $\exists Q = (M, \Upsilon) \in States(N)\ \exists t, u \in T_N$ s.t. ${}^\bullet t \cup {}^\bullet u \subseteq M,\ \Upsilon(t) = \Upsilon(u) = 0$ and $l_N(t) = l_N(u) = a$.

A time SM-refinement operation does not replace transitions labelled by conflicting or autoconcurrent actions, since it can cause problems similar to that of discussed in [6].

Definition 7.2 *Let* $N = \langle P_N, T_N, F_N, l_N, M_N, \Upsilon_N\rangle$ *be some time net,* $a \in l_N(T_N)$ *be neither conflicting nor autoconcurrent and* $D = \langle P_D, T_D, F_D, l_D, M_D, \Upsilon_D\rangle$ *be time SM-net. We define* $T = \{t \in T_N \mid (l_N(t) = a) \wedge (\Upsilon_N(t) = \Upsilon(D))\}$. *A* time SM-refinement, *denoted by* $tref(N, a, D)$, *is (up to isomorphism) a time net* $\overline{N} = \langle P_{\overline{N}}, T_{\overline{N}}, F_{\overline{N}}, l_{\overline{N}}, M_{\overline{N}}, \Upsilon_{\overline{N}}\rangle$, *where:*

- $P_{\overline{N}} = P_N \cup \{\langle p, u\rangle \mid p \in P_D \setminus \{p_{in}, p_{out}\},\ u \in T\}$;
- $T_{\overline{N}} = (T_N \setminus T) \cup \{\langle t, u\rangle \mid t \in T_D,\ u \in T\}$;
- $F_{\overline{N}}(\bar{x}, \bar{y}) = \begin{cases} F_N(\bar{x}, \bar{y}), & \bar{x}, \bar{y} \in P_N \cup (T_N \setminus T); \\ F_D(x, y), & \bar{x} = \langle x, u\rangle,\ \bar{y} = \langle y, u\rangle,\ u \in T; \\ F_N(\bar{x}, u), & \bar{y} = \langle y, u\rangle,\ \bar{x} \in {}^\bullet u,\ u \in T,\ y \in p_{in}^\bullet; \\ F_N(u, \bar{y}), & \bar{x} = \langle x, u\rangle,\ \bar{y} \in {}^\bullet u,\ u \in T,\ x \in {}^\bullet p_{out}; \\ 0, & otherwise; \end{cases}$
- $l_{\overline{N}}(\bar{u}) = \begin{cases} l_N(\bar{u}), & \bar{u} \in T_N \setminus T; \\ l_D(t), & \bar{u} = \langle t, u\rangle,\ t \in T_D,\ u \in T; \end{cases}$
- $M_{\overline{N}}(p) = M_N(p)$;
- $\Upsilon_{\overline{N}}(\bar{u}) = \begin{cases} \Upsilon_N(\bar{u}), & \bar{u} \in T_N \setminus T; \\ \Upsilon_D(t), & \bar{u} = \langle t, u\rangle,\ t \in T_D,\ u \in T. \end{cases}$

An equivalence is *preserved by refinements*, if equivalent nets remain equivalent after applying any refinement operator to them accordingly.

Example 7.1 *In Figure 5 a result of applying time SM-refinement operation is demonstrated.*

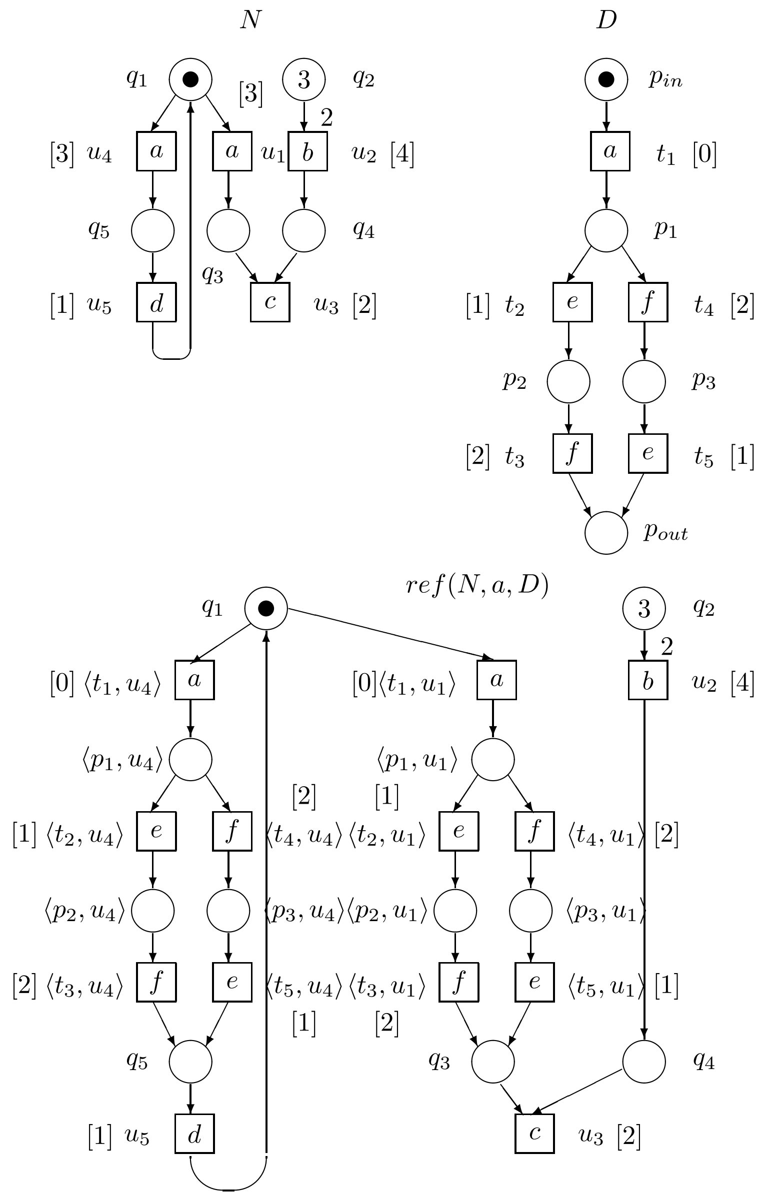

Figure 5: An example of time SM-refinement

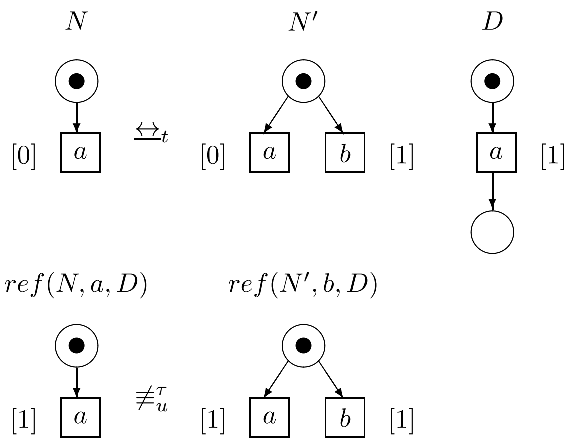

Figure 6: The equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_t$ are not preserved by SM-refinements

The following examples explain the need of time respect in SM-refinement operation and the requirement of replacing only transitions which are not labelled by conflictiing od autoconcurrent actions.

Example 7.2 • *In Figure 6 $N\underline{\leftrightarrow}_t N'$, but $ref(N, a, D) \not\equiv_u^\tau ref(N', a, D)$, since only in the time net $ref(N', a, D)$ an action b can happen. Hence, the equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_t$ (i.e. all with the exception of $\simeq$) are not preserved by SM-refinement.*

The reason is: an action b could not happen in N', since the choice was always in favour of a, and after refinement the initial time delay of all transitions labelled by a changed from 0 to 1 and became the same as for b-labelled transitions. Therefore, a refinement operation must respect time and replace transitions by time SM-nets with the same time delay only. Consequently, instead of ref we should use $tref$.

- *In Figure 7 $N\underline{\leftrightarrow}_t N'$, but $tref(N, a, D) \not\equiv_u^\tau tref(N', a, D)$, since only in the time net $ref(N', a, D)$ the sequence of actions a_1ba_2 cannot happen. Hence, the equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_t$ (i.e. all with the exception of $\simeq$) are not preserved by SM-refinement.*

  The reason is: the transition of N' with label a is in conflict, i.e. a is a conflicting action. Therefore, a time SM-refinement operation should not replace transitions labelled by conflicting actions.

- *In Figure 8 $N\underline{\leftrightarrow}_t N'$, but $tref(N, a, D) \not\equiv_u^\tau tref(N', a, D)$, since only in the time net $ref(N', a, D)$ the sequence of actions a_1a_1 cannot happen. Hence, the equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_t$ (i.e. all with the exception of $\simeq$) are not preserved by SM-refinement.*

  The reason is: in the time net N two transitions with label a are enabled, i.e. a is an autoconcurrent action. Therefore, a time SM-refinement operation should not replace transitions labelled by autoconcurrent actions.

Let us note that any SM-net may be combined from elementare SM-nets (consisting of one transition with the only input and the only output place) with use of alternative (choice) and sequential composition operations.

Hence, a refinement by general time SM-net may be replaced by sequence of *simple* time SM-refinements: *renaming*, *simple choice* and *simple splitting*, which substitute transitions by time SM-nets D_1, D_2 D_3, depicted in Figure 9 correspondently. Then the requirement 4 from general time SM-net is turned into the following conditions.

- For D_1 : $\Upsilon_{D_1}(t) = \Upsilon(D_1)$.
- For D_2 : $\Upsilon_{D_2}(t_1) = \Upsilon_{D_2}(t_2) = \Upsilon(D_2)$.
- For D_3 : $\Upsilon_{D_3}(t_1) + \Upsilon_{D_3}(t_2) = \Upsilon(D_3)$.

The following proposition demonstrates that all the considered trace timed equivalences and untimed equivalences are not preserved by time SM-refinements.

Proposition 7.1 *The equivalences $\equiv_t, \equiv_u, \underline{\leftrightarrow}_u, \equiv_t^\tau, \equiv_u^\tau, \underline{\leftrightarrow}_u^\tau$ are not preserved by time SM-refinements.*

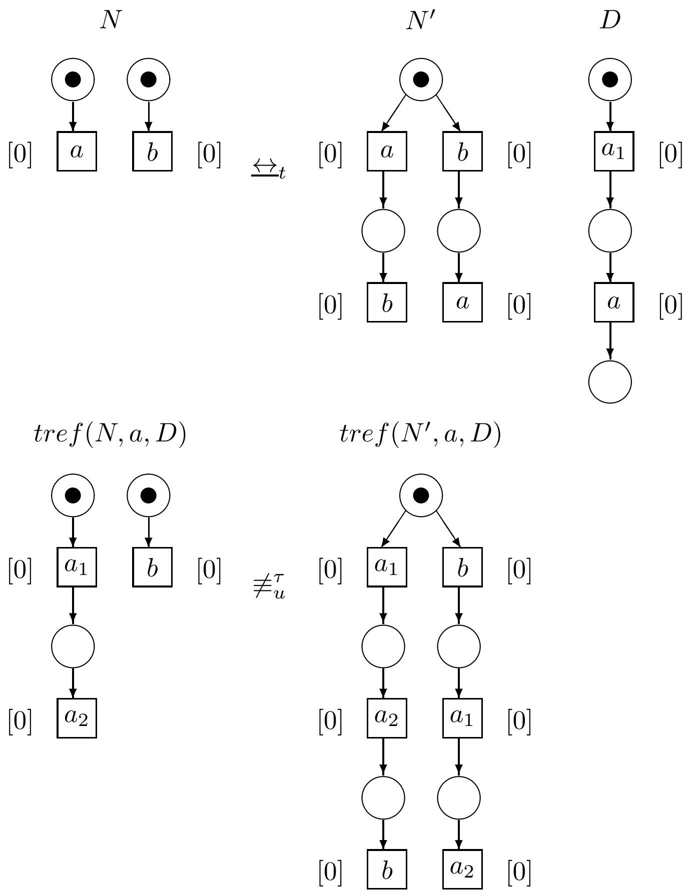

Figure 7: The equivalences between $\equiv^{\tau}_{u}$ and $\underline{\leftrightarrow}_{t}$ are not preserved by time SM-refinements (conflict)

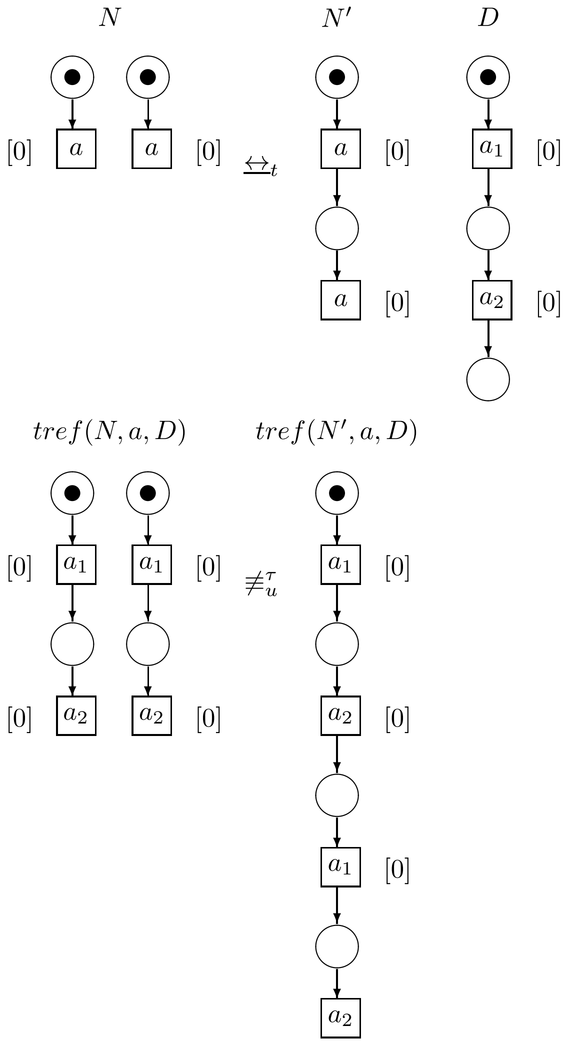

Figure 8: The equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_t$ are not preserved by time SM-refinements (autoconcurrency)

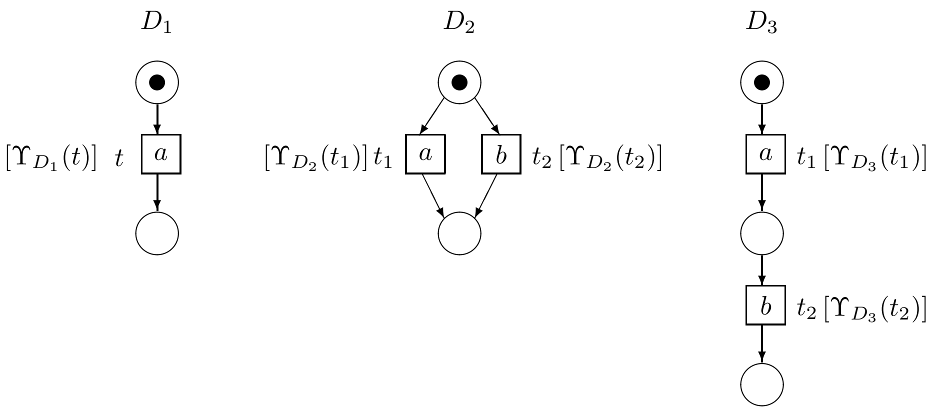

Figure 9: Time SM-nets of renaming, simple choice and simple splitting

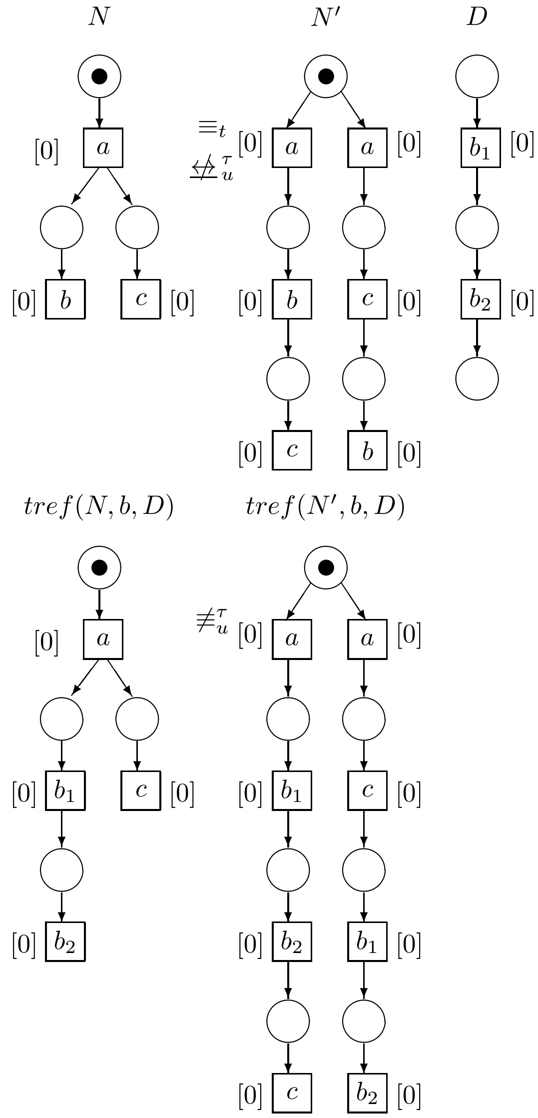

Figure 10: The equivalences between $\equiv_u^\tau$ and $\equiv_t$ are not preserved by time SM-refinements

Proof.

- In Figure 10 $N \equiv_t N'$, but $tref(N, b, D) \not\equiv_u^\tau tref(N', b, D)$, since only in the time net $tref(N', b, D)$ the sequence of actions ab_1c can never occur. Hence, the equivalences between $\equiv_u^\tau$ and $\equiv_t$ are not preserved by time SM-refinements.

- In Figure 11 $N \underline{\leftrightarrow}_u N'$, but $tref(N, a, D) \not\equiv_u^\tau tref(N', a, D)$, since only in the time net $tref(N', a, D)$ action b can happen. Hence, the equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_u$ are not preserved by time SM-refinements. □

The following theorem demonstrates which of the considered equivalences are preserved by time SM-refinements.

Theorem 7.1 *Let* $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star \in \{\_, t, u\}$. *For time nets* $N,\ N'$ *s.t.* $a \in l_N(T_N) \cap l_{N'}(T_{N'}) \cap Act$ *and time SM-net* D $N \leftrightarrow_\star N' \;\Rightarrow\; tref(N, a, D) \leftrightarrow_\star tref(N', a, D)$ *iff the equivalence* $\leftrightarrow_\star$ *is in oval in Figure 12.*

Proof. The proof is based ot the fact that in case of non-simulation of one net by another, one of initial time nets has conflicting or autoconcurrent actions, and it is in contradiction with definition of time SM-refinement. □

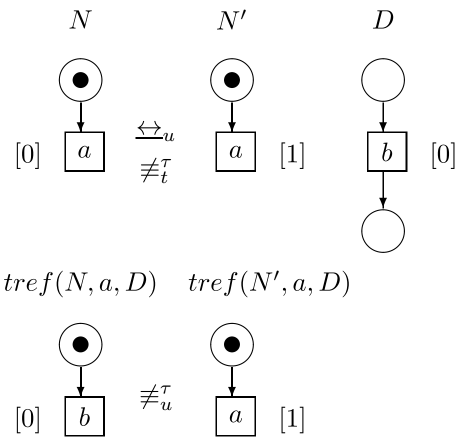

Figure 11: The equivalences between $\equiv_u^\tau$ and $\underline{\leftrightarrow}_u$ are not preserved by time SM-refinements

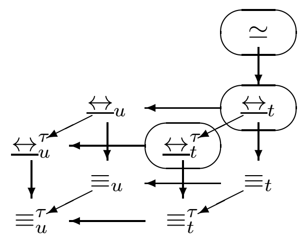

Figure 12: Preservation of the equivalences by time SM-refinements

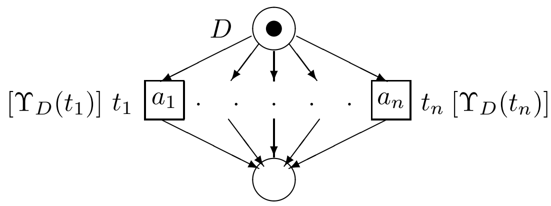

Figure 13: Time SM-net of n-choice

Let us note that result of the theorem is also valid for modification of time SM-refinement operation by letting to replace transitions with conflictiong and autoconcurrent actions by time SM-nets of a special kind: *n-choice*. An example of such a net is depicted in Figure 13. The condition for initial time delays of transitions is the following: $\forall i\ (1 \leq i \leq n)\ \Upsilon_D(t_i) = \Upsilon(D)$.

8 Conclusion

In this paper we investigated rather complete set of equivalences for time nets with silent actions and their subclass of untime nets. All the equivalence relations were compared on general time nets and their subclass of sequential time nets with each other and with regional equivalences which provides a possibility to simplify their check. The equivalence relations were treated for preservation by new operation of time SM-refinement, and two candidate, which may be useful for multilevel design, have been found: timed bisimulation equivalence ($\underline{\leftrightarrow}_t$) and timed τ-bisimulation equivalence ($\underline{\leftrightarrow}_t^\tau$). All these results provide us a basis for behavioural reasoning about concurrent systems with time delays with usage of such powerful formalism as time Petri nets. For a desisner, it is very important to have such set of equivalences, to choose the simplest appropriate viewpoint systems to be modelled .

Further development may consist in attempt to propose time equivalences not only in interleaving semantics (as it was done), but in step and (maybe) pomset one.

Moreother, it is worth extending the results obtained to time nets with interval time delays which are more experssive formalism than that of with fixed time delays.

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