

# Performance analysis of the dining philosophers system in *dtSPBC*

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**Abstract:** Algebra *dtsPBC* is a discrete time stochastic extension of finite Petri box calculus (*PBC*) enriched with iteration.

In this work, within *dtsPBC*, a method of modeling and performance evaluation based on stationary behaviour analysis for concurrent systems is outlined applied to the dining philosophers system.

**Keywords:** stochastic process algebra, Petri box calculus, discrete time, iteration, stationary behaviour, performance evaluation, dining philosophers system.

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## Introduction

### *Algebra $PBC$ and its extensions*

- *Petri box calculus  $PBC$*  [BDH92]
- *Time Petri box calculus  $tPBC$*  [Kou00]
- *Timed Petri box calculus  $TPBC$*  [MF00]
- *Stochastic Petri box calculus  $sPBC$*  [MVF01, MVCC03]
- *Ambient Petri box calculus  $APBC$*  [FM03]
- *Arc time Petri box calculus  $atPBC$*  [Nia05]
- *Generalized stochastic Petri box calculus  $gsPBC$*  [MVCR08]
- *Discrete time stochastic Petri box calculus  $dt sPBC$*  [Tar05, Tar06]
- *Discrete time stochastic and immediate Petri box calculus  $dt siPBC$*  [TMV10]

## Syntax

The *set of all finite multisets* over  $X$  is  $\mathbb{N}_f^X$ .

The *set of all subsets* of  $X$  is  $2^X$ .

$Act = \{a, b, \dots\}$  is the set of *elementary actions*.

$\widehat{Act} = \{\hat{a}, \hat{b}, \dots\}$  is the set of *conjugated actions (conjugates)* s.t.  $a \neq \hat{a}$  and  $\hat{\hat{a}} = a$ .

$\mathcal{A} = Act \cup \widehat{Act}$  is the set of *all actions*.

$\mathcal{L} = \mathbb{N}_f^{\mathcal{A}}$  is the set of *all multiactions*.

The *alphabet* of  $\alpha \in \mathcal{L}$  is  $\mathcal{A}(\alpha) = \{x \in \mathcal{A} \mid \alpha(x) > 0\}$ .

An *activity (stochastic multiaction)* is a pair  $(\alpha, \rho)$ , where  $\alpha \in \mathcal{L}$  and  $\rho \in (0; 1)$  is the *probability* of multiaction  $\alpha$ .

$\mathcal{SL}$  is the set of *all activities*.

The *alphabet* of  $(\alpha, \rho) \in \mathcal{SL}$  is  $\mathcal{A}(\alpha, \rho) = \mathcal{A}(\alpha)$ .

The **operations**: *sequential execution*  $;$ , *choice*  $[\ ]$ , *parallelism*  $\|$ , *relabeling*  $[f]$ , *restriction*  $rs$ , *synchronization*  $sy$  and *iteration*  $[**]$ .

Sequential execution and choice have the **standard** interpretation.

Parallelism **does not include synchronization unlike that in standard** process algebras.

Relabeling functions  $f : \mathcal{A} \rightarrow \mathcal{A}$  are bijections preserving conjugates:  $\forall x \in \mathcal{A} \ f(\hat{x}) = \widehat{f(x)}$ .

For  $\alpha \in \mathcal{L}$ , let  $f(\alpha) = \sum_{x \in \alpha} f(x)$ .

Restriction over an action  $a$ : any process behaviour containing  $a$  or its conjugate  $\hat{a}$  is not allowed.

Let  $\alpha, \beta \in \mathcal{L}$  be two multiactions s.t. for  $a \in Act$  we have  $a \in \alpha$  and  $\hat{a} \in \beta$  or  $\hat{a} \in \alpha$  and  $a \in \beta$ .

Synchronization of  $\alpha$  and  $\beta$  by  $a$  is  $\alpha \oplus_a \beta = \gamma$ :

$$\gamma(x) = \begin{cases} \alpha(x) + \beta(x) - 1, & x = a \text{ or } x = \hat{a}; \\ \alpha(x) + \beta(x), & \text{otherwise.} \end{cases}$$

In the **iteration**, the **initialization** subprocess is executed first,

then the **body** one is performed **zero or more times**, finally, the **termination** one is executed.

Static expressions specify the structure of processes.

**Definition 1** Let  $(\alpha, \rho) \in \mathcal{SL}$  and  $a \in Act$ . A static expression of *dtSPBC* is

$$E ::= (\alpha, \rho) \mid E;E \mid E[]E \mid E||E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E*E*E].$$

*StatExpr* is the set of all static expressions of *dtSPBC*.

**Definition 2** Let  $(\alpha, \rho) \in \mathcal{SL}$  and  $a \in Act$ . A regular static expression of *dtSPBC* is

$$E ::= (\alpha, \rho) \mid E;E \mid E[]E \mid E||E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E*D*E],$$

$$\text{where } D ::= (\alpha, \rho) \mid D;E \mid D[]D \mid D[f] \mid D \text{ rs } a \mid D \text{ sy } a \mid [D*D*E].$$

*RegStatExpr* is the set of all regular static expressions of *dtSPBC*.

Dynamic expressions specify the states of processes.

Dynamic expressions are combined from static ones annotated with upper or lower bars.

The *underlying static expression* of a dynamic one: removing all upper and lower bars.

**Definition 3** Let  $E \in \text{StatExpr}$  and  $a \in \text{Act}$ . A dynamic expression of *dtSPBC* is

$$G ::= \overline{E} \mid \underline{E} \mid G;E \mid E;G \mid G[]E \mid E[]G \mid G||G \mid G[f] \mid G \text{ rs } a \mid G \text{ sy } a \mid \\ [G*E*E] \mid [E*G*E] \mid [E*E*G].$$

*DynExpr* is the set of *all dynamic expressions* of *dtSPBC*.

A *regular dynamic expression*: its underlying static expression is regular.

*RegDynExpr* is the set of *all regular dynamic expressions* of *dtSPBC*.

## Operational semantics

### Inaction rules

Inaction rules: instantaneous structural transformations.

Let  $E, F, K \in \text{RegStatExpr}$  and  $a \in \text{Act}$ .

Inaction rules for overlined and underlined regular static expressions

$\overline{E};\overline{F} \Rightarrow \overline{E};\overline{F}$	$\underline{E};\overline{F} \Rightarrow E;\overline{F}$	$E;\underline{F} \Rightarrow \underline{E};\underline{F}$
$\overline{E}[]\overline{F} \Rightarrow \overline{E}[]\overline{F}$	$\overline{E}[]\overline{F} \Rightarrow E[]\overline{F}$	$\underline{E}[]\overline{F} \Rightarrow \underline{E}[]\overline{F}$
$E[]\underline{F} \Rightarrow \underline{E}[]\underline{F}$	$\overline{E}[]\overline{F} \Rightarrow \overline{E}[]\overline{F}$	$\underline{E}[]\underline{F} \Rightarrow \underline{E}[]\underline{F}$
$\overline{E}[f] \Rightarrow \overline{E}[f]$	$\underline{E}[f] \Rightarrow \underline{E}[f]$	$\overline{E} \text{ rs } a \Rightarrow \overline{E} \text{ rs } a$
$\underline{E} \text{ rs } a \Rightarrow \underline{E} \text{ rs } a$	$\overline{E} \text{ sy } a \Rightarrow \overline{E} \text{ sy } a$	$\underline{E} \text{ sy } a \Rightarrow \underline{E} \text{ sy } a$
$\overline{[E*F*K]} \Rightarrow [\overline{E}*F*K]$	$[\underline{E}*F*K] \Rightarrow [E*\overline{F}*K]$	$[E*\underline{F}*K] \Rightarrow [E*\overline{F}*K]$
$[E*\underline{F}*K] \Rightarrow [E*F*\overline{K}]$	$[E*F*\underline{K}] \Rightarrow \underline{[E*F*K]}$	



Let  $E, F \in RegStatExpr$ ,  $G, H, \tilde{G}, \tilde{H} \in RegDynExpr$  and  $a \in Act$ .

Inaction rules for arbitrary regular dynamic expressions

$\frac{G \Rightarrow \tilde{G}, \circ \in \{;, []\}}{G \circ E \Rightarrow \tilde{G} \circ E}$	$\frac{G \Rightarrow \tilde{G}, \circ \in \{;, []\}}{E \circ G \Rightarrow E \circ \tilde{G}}$	$\frac{G \Rightarrow \tilde{G}}{G \parallel H \Rightarrow \tilde{G} \parallel H}$	$\frac{H \Rightarrow \tilde{H}}{G \parallel H \Rightarrow G \parallel \tilde{H}}$	$\frac{G \Rightarrow \tilde{G}}{G[f] \Rightarrow \tilde{G}[f]}$
$\frac{G \Rightarrow \tilde{G}, \circ \in \{rs, sy\}}{G \circ a \Rightarrow \tilde{G} \circ a}$	$\frac{G \Rightarrow \tilde{G}}{[G * E * F] \Rightarrow [\tilde{G} * E * F]}$	$\frac{G \Rightarrow \tilde{G}}{[E * G * F] \Rightarrow [E * \tilde{G} * F]}$	$\frac{G \Rightarrow \tilde{G}}{[E * F * G] \Rightarrow [E * F * \tilde{G}]}$	

An *operative regular dynamic expression*  $G$ : no inaction rule can be applied to it.

$OpRegDynExpr$  is the set of *all operative regular dynamic expressions* of  $dtSPBC$ .

We shall consider regular expressions only and omit the word “regular”.

**Definition 4**  $\approx = (\Rightarrow \cup \Leftarrow)^*$  is the structural equivalence of dynamic expressions in  $dtSPBC$ .

$G$  and  $G'$  are *structurally equivalent*,  $G \approx G'$ , if they can be reached each from other by applying inaction rules in forward or backward direction.

## Action and empty loop rules

Action rules: execution of non-empty multisets of activities at a time step.

Empty loop rule: execution of the empty multiset of activities at a time step.

For  $\Gamma \in \mathcal{IN}_f^{\mathcal{SL}}$ , let  $f(\Gamma) = \sum_{(\alpha, \rho) \in \Gamma} (f(\alpha), \rho)$ .

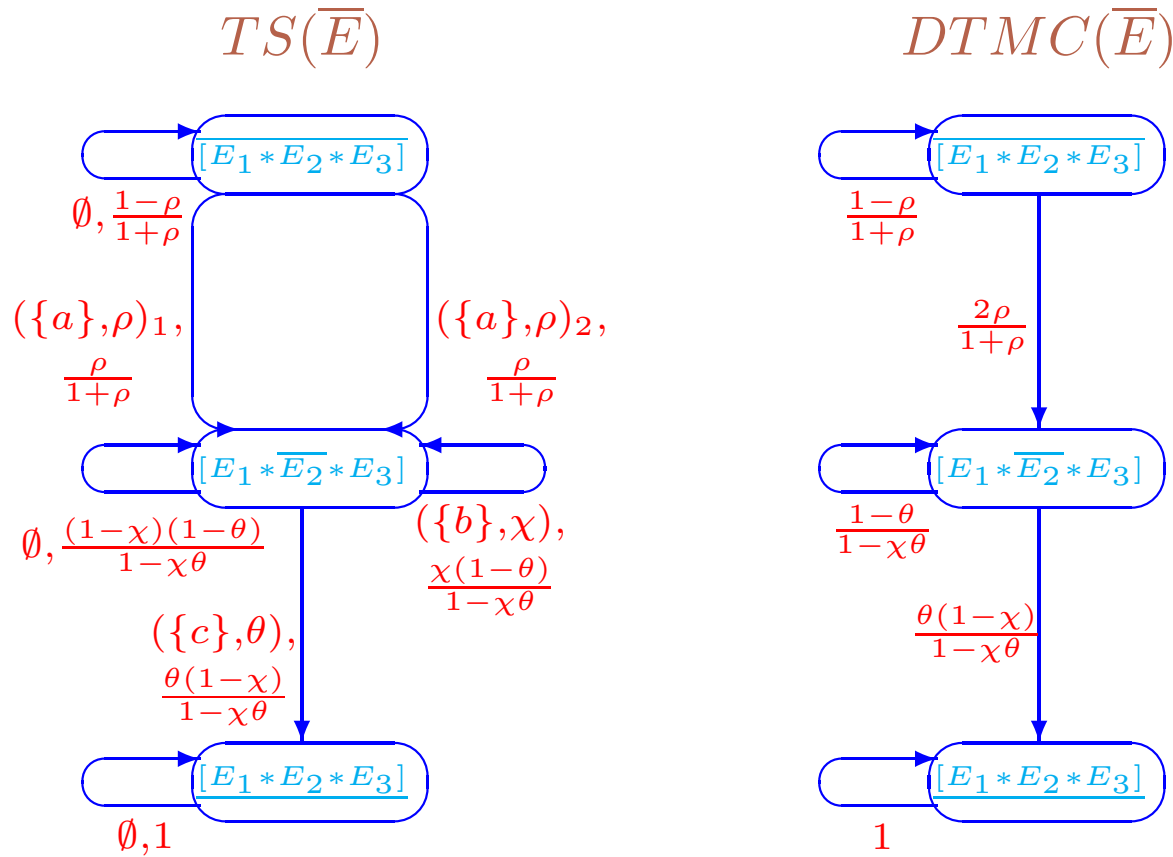
The *alphabet* of  $\Gamma \in \mathcal{IN}_f^{\mathcal{SL}}$  is  $\mathcal{A}(\Gamma) = \cup_{(\alpha, \rho) \in \Gamma} \mathcal{A}(\alpha)$ .

Let  $(\alpha, \rho), (\beta, \chi) \in \mathcal{SL}$ ,  $E, F \in \text{RegStatExpr}$ ,  $G, H \in \text{OpRegDynExpr}$ ,  $\tilde{G}, \tilde{H} \in \text{RegDynExpr}$ ,  $a \in \text{Act}$  and  $\Gamma, \Delta \in \mathcal{IN}_f^{\mathcal{SL}} \setminus \{\emptyset\}$ ,  $\Gamma' \in \mathcal{IN}_f^{\mathcal{SL}}$ .

### Action and empty loop rules

<b>E</b> $G \xrightarrow{\emptyset} G$	<b>B</b> $\overline{(\alpha, \rho)} \xrightarrow{\{(\alpha, \rho)\}} (\alpha, \rho)$	<b>SC1</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}, \circ \in \{;, []\}}{G \circ E \xrightarrow{\Gamma} \tilde{G} \circ E}$
<b>SC2</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}, \circ \in \{;, []\}}{E \circ G \xrightarrow{\Gamma} E \circ \tilde{G}}$	<b>P1</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{G \parallel H \xrightarrow{\Gamma} \tilde{G} \parallel H}$	<b>P2</b> $\frac{H \xrightarrow{\Gamma} \tilde{H}}{G \parallel H \xrightarrow{\Gamma} G \parallel \tilde{H}}$
<b>P3</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}, H \xrightarrow{\Delta} \tilde{H}}{G \parallel H \xrightarrow{\Gamma + \Delta} \tilde{G} \parallel \tilde{H}}$	<b>L</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{G[f] \xrightarrow{f(\Gamma)} \tilde{G}[f]}$	<b>RS</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}, a, \hat{a} \notin \mathcal{A}(\Gamma)}{G \text{ rs } a \xrightarrow{\Gamma} \tilde{G} \text{ rs } a}$
<b>I1</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{[G * E * F] \xrightarrow{\Gamma} [\tilde{G} * E * F]}$	<b>I2</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{[E * G * F] \xrightarrow{\Gamma} [E * \tilde{G} * F]}$	<b>I3</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{[E * F * G] \xrightarrow{\Gamma} [E * F * \tilde{G}]}$
<b>Sy1</b> $\frac{G \xrightarrow{\Gamma} \tilde{G}}{G \text{ sy } a \xrightarrow{\Gamma} \tilde{G} \text{ sy } a}$	<b>Sy2</b> $\frac{G \text{ sy } a \xrightarrow{\Gamma' + \{(\alpha, \rho)\} + \{(\beta, \chi)\}} \tilde{G} \text{ sy } a, a \in \alpha, \hat{a} \in \beta}{G \text{ sy } a \xrightarrow{\Gamma' + \{(\alpha \oplus_a \beta, \rho \cdot \chi)\}} \tilde{G} \text{ sy } a}$	

## Transition systems



**EXPRIT:** The transition system and the underlying DTMC of  $\bar{E}$  for  $E = [((\{a\}, \rho)_1 \parallel (\{a\}, \rho)_2) * (\{b\}, \chi) * (\{c\}, \theta)]$

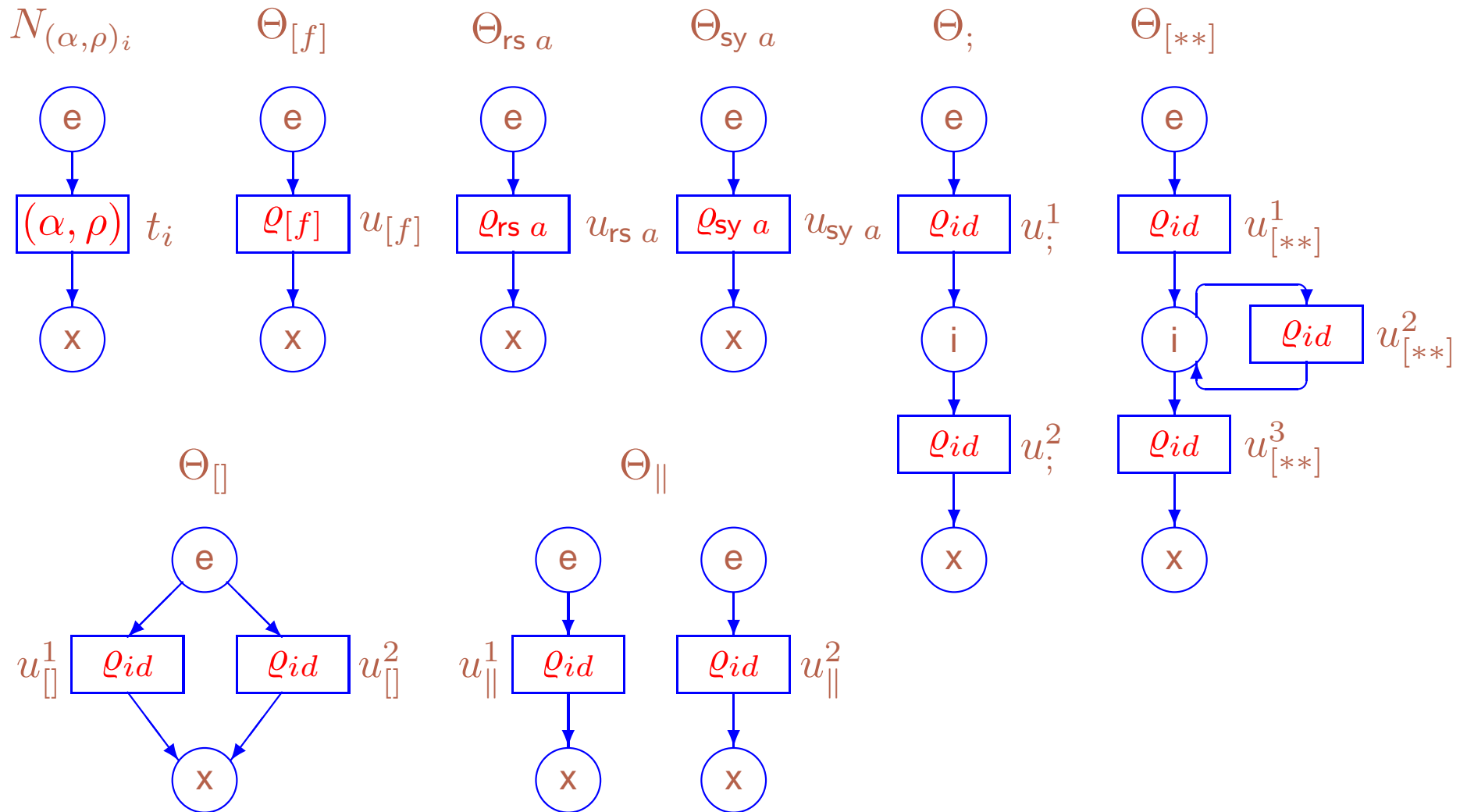
Let  $E_1 = (\{a\}, \rho) \parallel (\{a\}, \rho)$ ,  $E_2 = (\{b\}, \chi)$ ,  $E_3 = (\{c\}, \theta)$  and  $E = [E_1 * E_2 * E_3]$ .

The identical activities of the composite static expression are **enumerated** as:

$E = [((\{a\}, \rho)_1 \parallel (\{a\}, \rho)_2) * (\{b\}, \chi) * (\{c\}, \theta)]$ . The derivation set  $DR(\bar{E})$  of  $\bar{E}$  consists of

$s_1 = \overline{[E_1 * E_2 * E_3]} \approx$ ,  $s_2 = \overline{[E_1 * \bar{E}_2 * E_3]} \approx$ ,  $s_3 = \overline{[E_1 * E_2 * E_3]} \approx$ .

## Denotational semantics



The plain and operator dts-boxes

**Definition 5** Let  $(\alpha, \rho) \in \mathcal{SL}$ ,  $a \in \text{Act}$  and  $E, F, K \in \text{RegStatExpr}$ . The **denotational semantics** of *dtsPBC* is a mapping  $\text{Box}_{dts}$  from *RegStatExpr* into plain *dts*-boxes:

1.  $\text{Box}_{dts}((\alpha, \rho)_i) = N_{(\alpha, \rho)_i}$ ;
2.  $\text{Box}_{dts}(E \circ F) = \Theta_{\circ}(\text{Box}_{dts}(E), \text{Box}_{dts}(F))$ ,  $\circ \in \{;, [], \|\}$ ;
3.  $\text{Box}_{dts}(E[f]) = \Theta_{[f]}(\text{Box}_{dts}(E))$ ;
4.  $\text{Box}_{dts}(E \circ a) = \Theta_{\circ a}(\text{Box}_{dts}(E))$ ,  $\circ \in \{\text{rs}, \text{sy}\}$ ;
5.  $\text{Box}_{dts}([E * F * K]) = \Theta_{[**]}(\text{Box}_{dts}(E), \text{Box}_{dts}(F), \text{Box}_{dts}(K))$ .

For  $E \in \text{RegStatExpr}$ , let  $\text{Box}_{dts}(\overline{E}) = \overline{\text{Box}_{dts}(E)}$  and  $\text{Box}_{dts}(\underline{E}) = \underline{\text{Box}_{dts}(E)}$ .

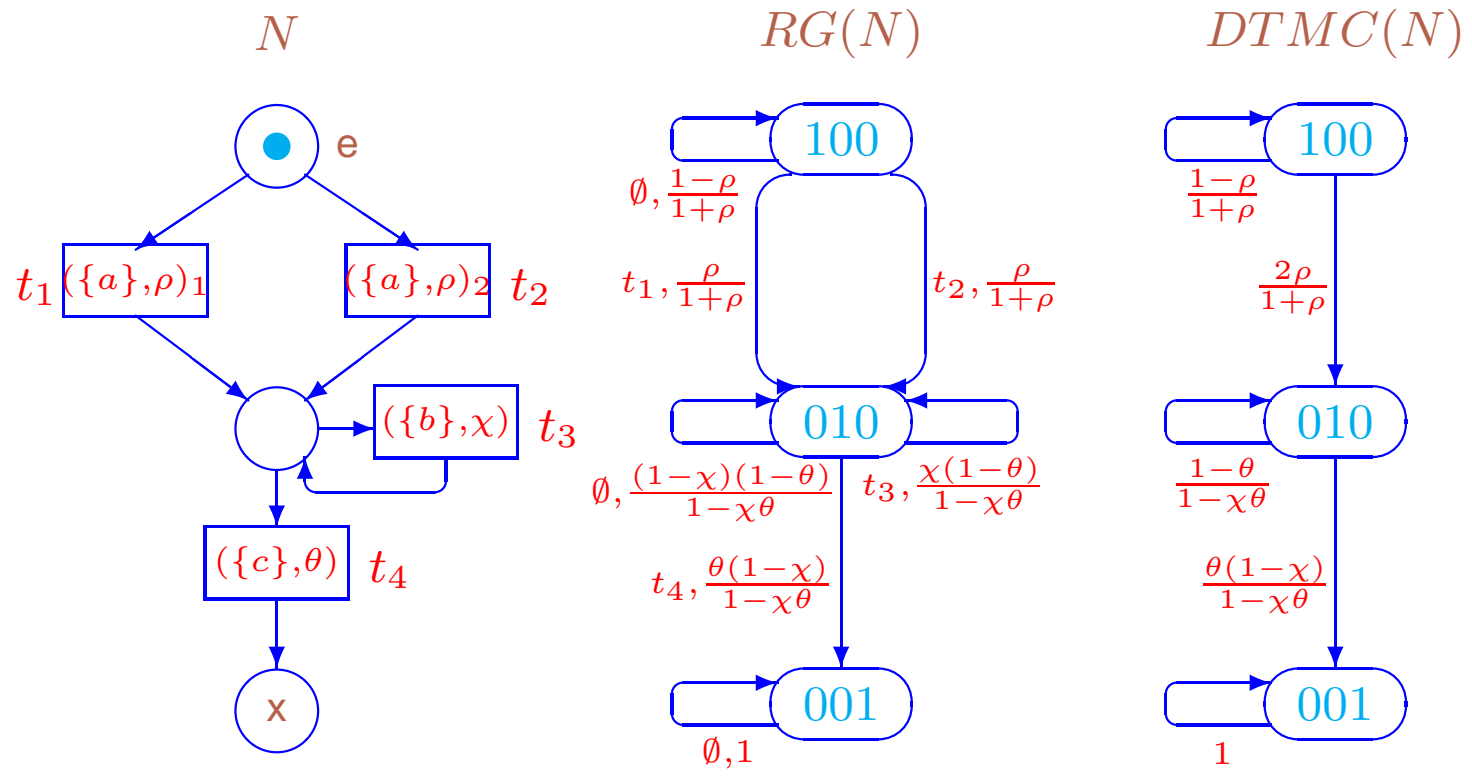
We denote isomorphism of transition systems by  $\simeq$ ,

and **the same symbol** denotes isomorphism of reachability graphs and DTMCs

as well as isomorphism between transition systems and reachability graphs.

**Theorem 1** For any static expression  $E$  we have  $TS(\overline{E}) \simeq RG(\text{Box}_{dts}(\overline{E}))$ .

**Proposition 1** For any static expression  $E$  we have  $DTMC(\overline{E}) \simeq DTMC(\text{Box}_{dts}(\overline{E}))$ .



**BOXIT:** The marked dts-box  $N = \text{Box}_{dts}(\overline{E})$  for  $E = [((\{a\}, \rho)_1 [(\{a\}, \rho)_2] * (\{b\}, \chi) * (\{c\}, \theta))$ , its reachability graph and the underlying DTMC

## Performance evaluation

The elements  $\mathcal{P}_{ij}$  ( $1 \leq i, j \leq n = |DR(G)|$ ) of *(one-step) transition probability matrix (TPM)*  $\mathbf{P}$  for *DTMC*( $G$ ):

$$\mathcal{P}_{ij} = \begin{cases} PM(s_i, s_j), & s_i \rightarrow s_j; \\ 0, & \text{otherwise.} \end{cases}$$

The *transient* ( $k$ -step,  $k \in \mathbb{N}$ ) *probability mass function (PMF)*  $\psi[k] = (\psi_1[k], \dots, \psi_n[k])$  for *DTMC*( $G$ ) is the solution of  $\psi[k] = \psi[0]\mathbf{P}^k$ ,

where  $\psi[0] = (\psi_1[0], \dots, \psi_n[0])$  is the *initial PMF*:  $\psi_i[0] = \begin{cases} 1, & s_i = [G]_{\approx}; \\ 0, & \text{otherwise.} \end{cases}$

We have  $\psi[k+1] = \psi[k]\mathbf{P}$ ,  $k \in \mathbb{N}$ .

The *steady-state PMF*  $\psi = (\psi_1, \dots, \psi_n)$  for *DTMC*( $G$ ) is the solution of  $\begin{cases} \psi(\mathbf{P} - \mathbf{E}) = \mathbf{0} \\ \psi\mathbf{1}^T = 1 \end{cases}$ ,

where  $\mathbf{0}$  is a vector with  $n$  values 0,  $\mathbf{1}$  is that with  $n$  values 1.

When *DTMC*( $G$ ) has the single steady state,  $\psi = \lim_{k \rightarrow \infty} \psi[k]$ .

For  $s \in DR(G)$  with  $s = s_i$  ( $1 \leq i \leq n$ ) we define  $\psi[k](s) = \psi_i[k]$  ( $k \in \mathbb{N}$ ) and  $\psi(s) = \psi_i$ .

Let  $G$  be a dynamic expression and  $s, \tilde{s} \in DR(G)$ ,  $S, \tilde{S} \subseteq DR(G)$ .

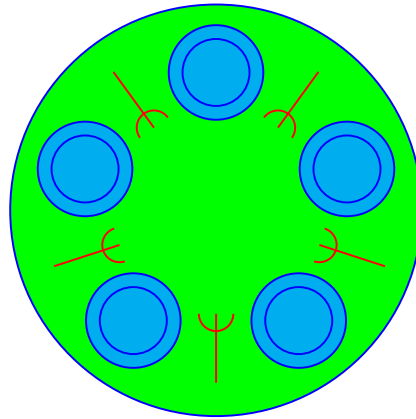
The following **performance indices (measures)** are based on the steady-state PMF.

- The **average recurrence (return) time in the state  $s$**  (the number of discrete time units or steps required for this) is  $\frac{1}{\psi(s)}$ .
- The **fraction of residence time in the state  $s$**  is  $\psi(s)$ .
- The **fraction of residence time in the set of states  $S \subseteq DR(G)$**  or the **probability of the event determined by a condition that is true for all states from  $S$**  is  $\sum_{s \in S} \psi(s)$ .
- The **relative fraction of residence time in the set of states  $S$  w.r.t. that in  $\tilde{S}$**  is  $\frac{\sum_{s \in S} \psi(s)}{\sum_{\tilde{s} \in \tilde{S}} \psi(\tilde{s})}$ .
- The **steady-state probability to perform a step with an activity  $(\alpha, \rho)$**  is  $\sum_{s \in DR(G)} \psi(s) \sum_{\{\Gamma | (\alpha, \rho) \in \Gamma\}} PT(\Gamma, s)$ .
- The **probability of the event determined by a reward function  $r$  on the states** is  $\sum_{s \in DR(G)} \psi(s) r(s)$ .



## Dining philosophers system

A model of five dining philosophers [P81]



The diagram of the dining philosophers system

After activation of the system, five forks appear on the table.

If the left and right forks available for a philosopher, he takes them simultaneously and begins eating.

At the end of eating, the philosopher places both his forks simultaneously back on the table.

$a$  corresponds to the system activation.

$b_i$  and  $e_i$  correspond to the beginning and the end of eating of philosopher  $i$  ( $1 \leq i \leq 5$ ).

The other actions are used for communication purpose only.

The expression of each philosopher includes two alternative subexpressions:

the second one specifies a resource (fork) sharing with the right neighbor.

The static expression of the philosopher  $i$  ( $1 \leq i \leq 4$ ) is

$$E_i = [(\{x_i\}, \frac{1}{2}) * (((\{b_i, \widehat{y}_i\}, \frac{1}{2}); (\{e_i, \widehat{z}_i\}, \frac{1}{2})) \square ((\{y_{i+1}\}, \frac{1}{2}); (\{z_{i+1}\}, \frac{1}{2}))) * \text{Stop}].$$

The static expression of the philosopher 5 is

$$E_5 = [(\{a, \widehat{x}_1, \widehat{x}_2, \widehat{x}_2, \widehat{x}_4\}, \frac{1}{2}) * (((\{b_5, \widehat{y}_5\}, \frac{1}{2}); (\{e_5, \widehat{z}_5\}, \frac{1}{2})) \square ((\{y_1\}, \frac{1}{2}); (\{z_1\}, \frac{1}{2}))) * \text{Stop}].$$

The static expression of the dining philosophers system is

$$E = (E_1 \parallel E_2 \parallel E_3 \parallel E_4 \parallel E_5) \text{ sy } x_1 \text{ sy } x_2 \text{ sy } x_3 \text{ sy } x_4 \text{ sy } y_1 \text{ sy } y_2 \text{ sy } y_3 \text{ sy } y_4 \text{ sy } y_5 \text{ sy } z_1 \text{ sy } z_2 \\ \text{ sy } z_3 \text{ sy } z_4 \text{ sy } z_5 \text{ rs } x_1 \text{ rs } x_2 \text{ rs } x_3 \text{ rs } x_4 \text{ rs } y_1 \text{ rs } y_2 \text{ rs } y_3 \text{ rs } y_4 \text{ rs } y_5 \text{ rs } z_1 \text{ rs } z_2 \text{ rs } z_3 \text{ rs } z_4 \text{ rs } z_5.$$

Interpretation of the states

$s_1$ : the initial state,

$s_2$ : the system is activated and no philosophers dine,

$s_3$ : philosopher 1 dines,

$s_4$ : philosophers 1 and 4 dine,

$s_5$ : philosophers 1 and 3 dine,

$s_6$ : philosopher 4 dines,

$s_7$ : philosopher 3 dines,

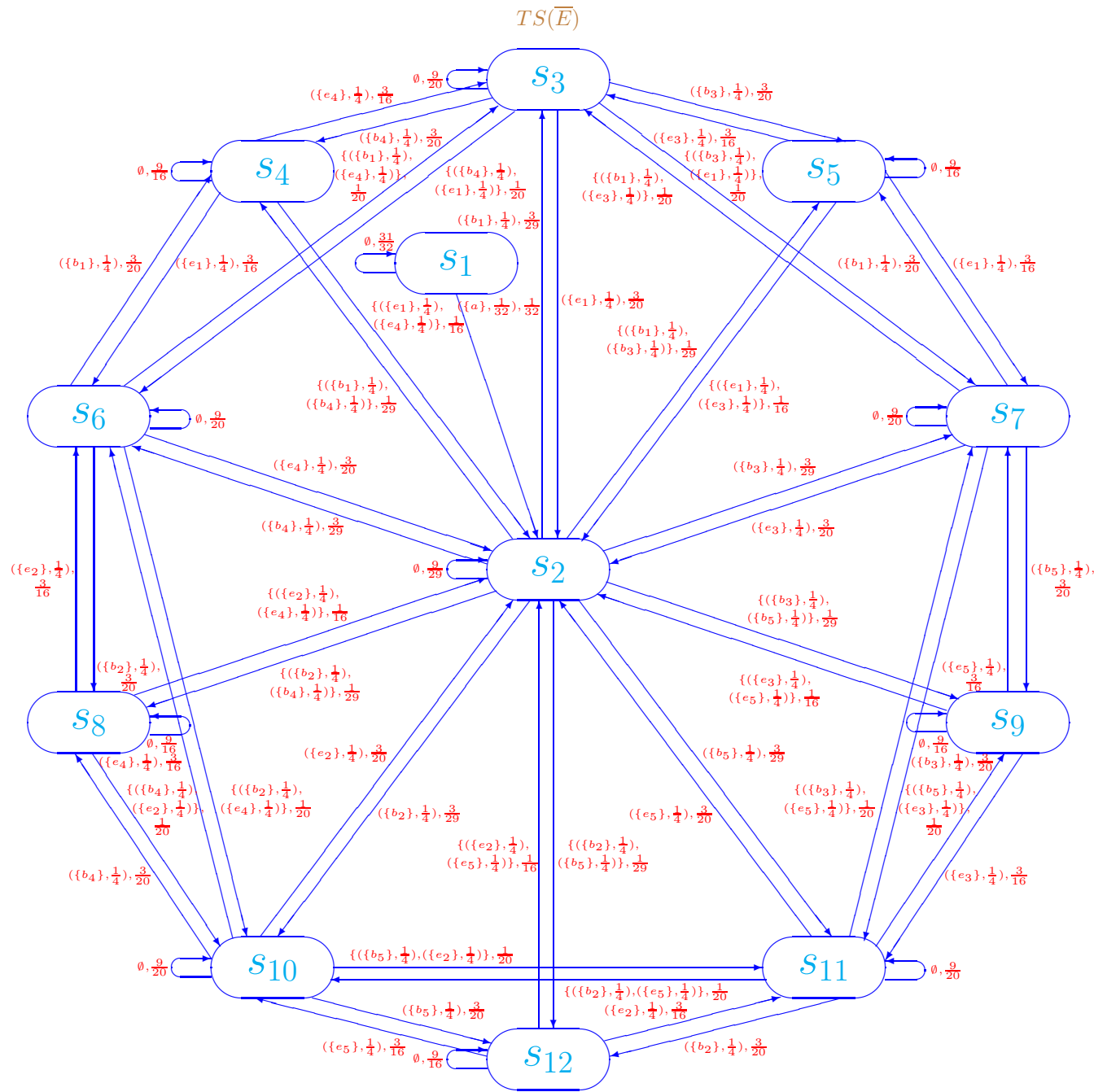
$s_8$ : philosophers 2 and 4 dine,

$s_9$ : philosophers 3 and 5 dine,

$s_{10}$ : philosopher 2 dines,

$s_{11}$ : philosopher 5 dine,

$s_{12}$ : philosophers 2 and 5 dine.



The transition system of the dining philosophers system

The TPM for  $DTMC^*(\bar{E})$  is

$$\mathbf{P} = \begin{bmatrix} \frac{31}{32} & \frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{9}{29} & \frac{3}{29} & \frac{1}{29} & \frac{1}{29} & \frac{3}{29} & \frac{3}{29} & \frac{1}{29} & \frac{1}{29} & \frac{3}{29} & \frac{3}{29} & \frac{1}{29} \\ 0 & \frac{3}{20} & \frac{9}{20} & \frac{3}{20} & \frac{3}{20} & \frac{1}{20} & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{16} & \frac{3}{16} & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{16} & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{20} & \frac{1}{20} & \frac{3}{20} & 0 & \frac{9}{20} & 0 & \frac{3}{20} & 0 & \frac{1}{20} & 0 & 0 \\ 0 & \frac{3}{20} & \frac{1}{20} & 0 & \frac{3}{20} & 0 & \frac{9}{20} & 0 & \frac{3}{20} & 0 & \frac{1}{20} & 0 \\ 0 & \frac{1}{16} & 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 \\ 0 & \frac{1}{16} & 0 & 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 \\ 0 & \frac{3}{20} & 0 & 0 & 0 & \frac{1}{20} & 0 & \frac{3}{20} & 0 & \frac{9}{20} & \frac{1}{20} & \frac{3}{20} \\ 0 & \frac{3}{20} & 0 & 0 & 0 & 0 & \frac{1}{20} & 0 & \frac{3}{20} & \frac{1}{20} & \frac{9}{20} & \frac{3}{20} \\ 0 & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{16} & \frac{3}{16} & \frac{9}{16} \end{bmatrix}.$$

The average sojourn time vector of  $\overline{E}$  is

$$SJ = \left( 32, \frac{29}{20}, \frac{20}{11}, \frac{16}{7}, \frac{16}{7}, \frac{20}{11}, \frac{20}{11}, \frac{16}{7}, \frac{16}{7}, \frac{20}{11}, \frac{20}{11}, \frac{16}{7} \right).$$

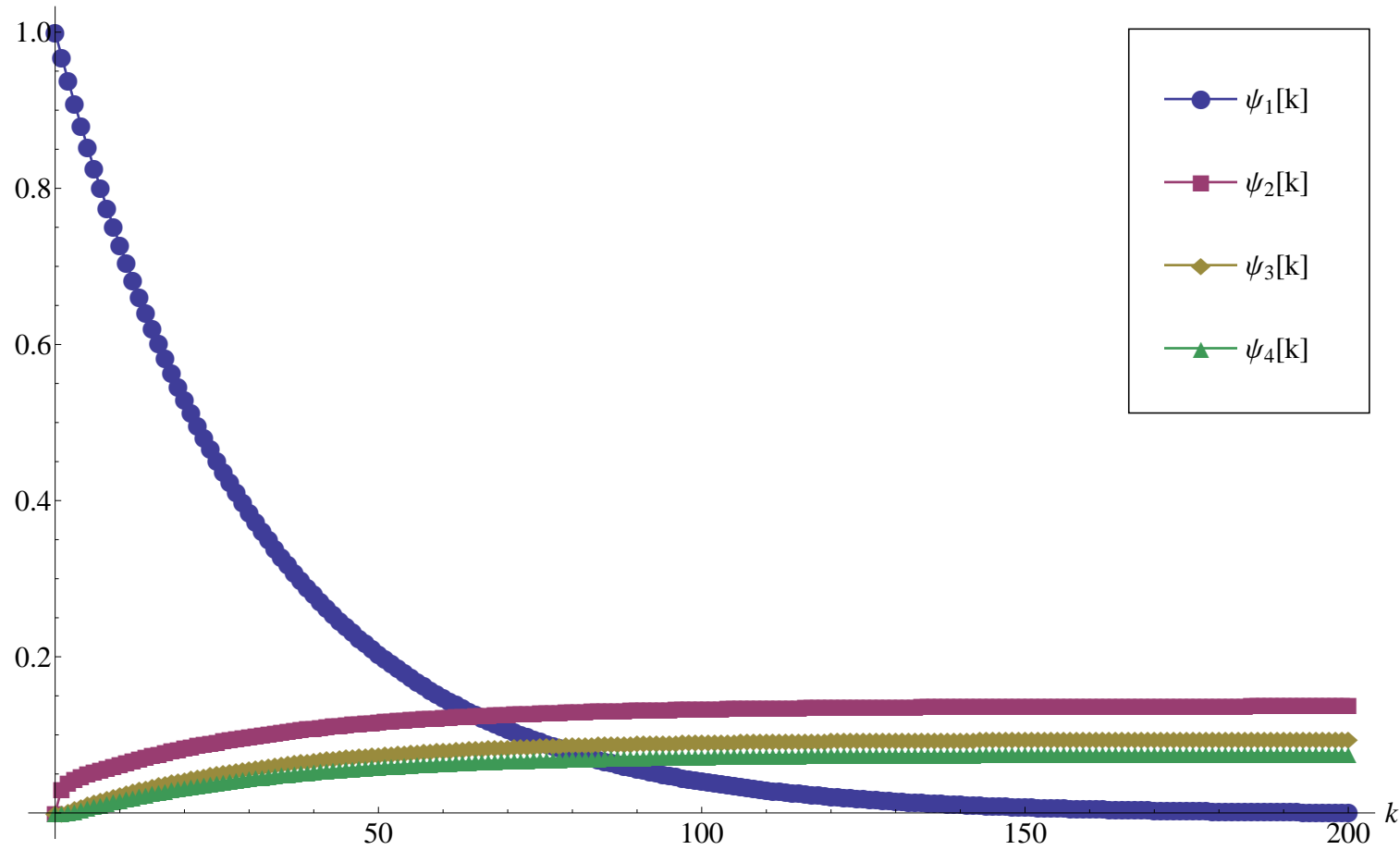
The sojourn time variance vector of  $\overline{E}$  is

$$VAR = \left( 1024, \frac{841}{400}, \frac{400}{121}, \frac{256}{49}, \frac{256}{49}, \frac{400}{121}, \frac{400}{121}, \frac{256}{49}, \frac{256}{49}, \frac{400}{121}, \frac{400}{121}, \frac{256}{49} \right).$$

### Transient and steady-state probabilities of the dining philosophers system

$k$	0	20	20	60	80	100	120	140	160	180	200	$\infty$
$\psi_1[k]$	1	0.5299	0.2808	0.1488	0.0789	0.0418	0.0222	0.0117	0.0062	0.0033	0.0017	0
$\psi_2[k]$	0	0.0842	0.1098	0.1234	0.1306	0.1345	0.1365	0.1375	0.1381	0.1384	0.1386	0.1388
$\psi_3[k]$	0	0.0437	0.0681	0.0811	0.0880	0.0916	0.0935	0.0945	0.0951	0.0954	0.0955	0.0957
$\psi_4[k]$	0	0.0335	0.0537	0.0645	0.0701	0.0732	0.0748	0.0756	0.0760	0.0763	0.0764	0.0766

We depict the probabilities for the states  $s_1, \dots, s_4$  only, since the corresponding values coincide for the states  $s_3, s_6, s_7, s_{10}, s_{11}$  as well as for  $s_4, s_5, s_8, s_9, s_{12}$ .



Transient probabilities alteration diagram of the dining philosophers system

The steady-state PMF for  $DTMC^*(\bar{E})$  is

$$\psi = \left( 0, \frac{29}{209}, \frac{20}{209}, \frac{16}{209}, \frac{16}{209}, \frac{20}{209}, \frac{20}{209}, \frac{16}{209}, \frac{16}{209}, \frac{20}{209}, \frac{20}{209}, \frac{16}{209} \right).$$

## Performance indices

- The average recurrence time in the state  $s_2$ , where all the forks are available, the *average system run-through*, is  $\frac{1}{\psi_2} = \frac{209}{29} = 7\frac{6}{29}$ .

- Nobody eats in the state  $s_2$ . The *fraction of time when no philosophers dine* is  $\psi_2 = \frac{29}{209}$ .

Only one philosopher eats in the states  $s_3, s_6, s_7, s_{10}, s_{11}$ . The *fraction of time when only one philosopher dines* is  $\psi_3 + \psi_6 + \psi_7 + \psi_{10} + \psi_{11} = \frac{20}{209} + \frac{20}{209} + \frac{20}{209} + \frac{20}{209} + \frac{20}{209} = \frac{100}{209}$ .

Two philosophers eat together in the states  $s_4, s_5, s_8, s_9, s_{12}$ . The *fraction of time when two philosophers dine* is  $\psi_4 + \psi_5 + \psi_8 + \psi_9 + \psi_{12} = \frac{16}{209} + \frac{16}{209} + \frac{16}{209} + \frac{16}{209} + \frac{16}{209} = \frac{80}{209}$ .

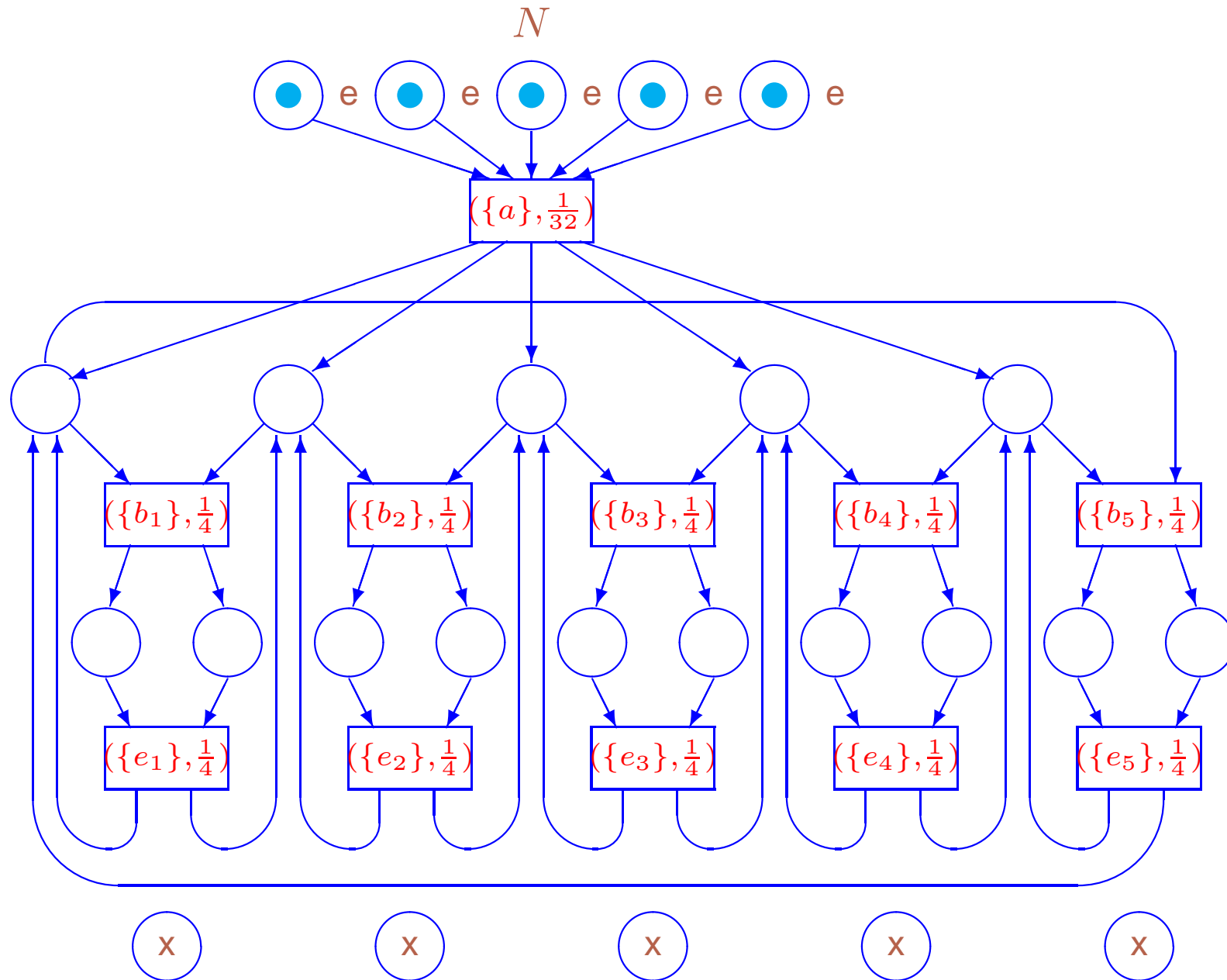
The *relative fraction of time when two philosophers dine w.r.t. when only one philosopher dines* is  $\frac{80}{209} \cdot \frac{209}{100} = \frac{4}{5}$ .

- The beginning of eating of first philosopher ( $\{b_1\}, \frac{1}{4}$ ) is only possible from the states  $s_2, s_6, s_7$ .

The beginning of eating probability in each of the states is a sum of execution probabilities for all multisets of activities containing ( $\{b_1\}, \frac{1}{4}$ ).

The *steady-state probability of the beginning of eating of first philosopher* is

$$\begin{aligned} & \psi_2 \sum_{\{\Gamma | (\{b_1\}, \frac{1}{4}) \in \Gamma\}} PT(\Gamma, s_2) + \psi_6 \sum_{\{\Gamma | (\{b_1\}, \frac{1}{4}) \in \Gamma\}} PT(\Gamma, s_6) + \\ & \psi_7 \sum_{\{\Gamma | (\{b_1\}, \frac{1}{4}) \in \Gamma\}} PT(\Gamma, s_7) = \\ & \frac{29}{209} \left( \frac{3}{29} + \frac{1}{29} + \frac{1}{29} \right) + \frac{20}{209} \left( \frac{3}{20} + \frac{1}{20} \right) + \frac{20}{209} \left( \frac{3}{20} + \frac{1}{20} \right) = \frac{13}{209}. \end{aligned}$$



The marked dts-box of the dining philosophers system



## Overview and open questions

### The results obtained

- A discrete time stochastic extension  $dt sPBC$  of finite  $PBC$  enriched with iteration.
- A case study of performance analysis: the dining philosophers system.

### Further research

- Defining stochastic equivalences to identify stochastic processes with similar behaviour.
- Extending the syntax with recursion operator.

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