Report of Dr. Igor V. Tarasyuk on the Russian-German research project "Comparative Analysis and Verification for Concurrent Correctness-Critical Systems (CAVER)", supported by RFBR (14-01-91334) and DFG (BE 1267/14-1)

1.6. List of the most important publications resulting from this project.

(a) Articles which at the time of proposal submission have been published or officially accepted by publication (up to three main publications per each year are marked by the symbol '').*

[TMV14a] Tarasyuk I.V., Macia S.H., Valero R.V. Performance analysis of concurrent systems in algebra dtsiPBC. Programming 40(5), pages 3-27, MAIK Nauka / Interperiodica, Moscow, Russia, September 2014 (ISSN 0132-3474, in Russian), http://itar.iis.nsk.su/files/itar/pages/dtsialtpro.pdf.

*[TMV14b] Tarasyuk I.V., Macia S.H., Valero R.V. Performance analysis of concurrent systems in algebra dtsiPBC. Programming and Computer Software 40(5), pages 229-249, Pleiades Publishing, Ltd., September 2014 (ISSN 0361-7688), DOI: 10.1134/S0361768814050089, http://www.maik.rssi.ru/journals/procom.htm. Web of Science, Springer, Scopus, Zentralblatt Math indexed. JCR impact factor (2014): 0.191. SJR indicator (2014): 0.308.

*[TMV15] Tarasyuk I.V., Macia S.H., Valero R.V. Stochastic process reduction for performance evaluation in dtsiPBC. Siberian Electronic Mathematical Reports 12, pages 513-551, Sobolev Institute of Mathematics, Novosibirsk, Russia, September 2015 (ISSN 1813-3304), DOI: 10.17377/semi.2015.12.044, http://itar.iis.nsk.su/files/itar/pages/dtsipbcsemr.pdf, http://semr.math.nsc.ru/v12/p513-551.pdf. Web of Science, Scopus, Zentralblatt Math indexed. SJR indicator (2015): 0.415.

*[TB15] Tarasyuk I.V., Buchholz P. Bisimulation for fluid stochastic Petri nets. Bulletin of the Novosibirsk Computing Center, Series Computer Science, IIS Special Issue 38, pages 121-149, NCC Publisher, Novosibirsk, Russia, 2015 (ISSN 1680-6972), http://itar.iis.nsk.su/files/itar/pages/fspneqncc.pdf, http://bulletin.iis.nsk.su/files/article/fspneqncc.pdf. Zentralblatt Math indexed.

*[MVCRT16] Macia S.H., Valero R.V., Cuartero G.F., Ruiz M.C., Tarasyuk I.V. Modelling a video conference system with sPBC. Applied Mathematics and Information Sciences 10(2), pages 475-493, Natural Sciences Publishing, New York, NY, USA, March 2016 (ISSN 1935-0090). DOI: 10.18576/amis/100210. Scopus, Zentralblatt Math indexed. SJR indicator (2015): 0.350.

[TMV16] Tarasyuk I.V., Macia S.H., Valero R.V. Bisimulation equivalence and performance analysis of concurrent systems with discrete stochastic time in dtsiPBC. Technical Report DIAB-16-03-1, 92 pages, Department of Computer Systems, High School of Computer Science Engineering, University of Castilla - La Mancha, Albacete, Spain, March 2016, http://itar.iis.nsk.su/files/itar/pages/dtsipbctcstr.pdf, http://www.dsi.uclm.es/descargas/technicalreports/DIAB-16-03-1/dtsipbclmcs_tr_032016.pdf.

[TMV17] Tarasyuk I.V., Macia S.H., Valero R.V. Stochastic equivalence for performance analysis of concurrent systems in dtsiPBC. CoRR abs/1702.07478 (arXiv:1702.07478), 69 pages, Computing Research Repository, Cornell University Library, Ithaca, NY, USA, February 2017, http://itar.iis.nsk.su/files/itar/pages/dtsipbcarxiv.pdf, http://arxiv.org/pdf/1702.07478.pdf.

*[TB17a] Tarasyuk I.V., Buchholz P. Equivalences for fluid stochastic Petri nets. Siberian Electronic Mathematical Reports 14, pages 317-366, S.L. Sobolev Institute of Mathematics, Novosibirsk, Russia, April 2017 (ISSN 1813-3304), DOI: 10.17377/semi.2017.14.029, http://itar.iis.nsk.su/files/itar/pages/fspneqsemr.pdf, http://semr.math.nsc.ru/v14/p317-366.pdf. Web of Science, Scopus, Zentralblatt Math indexed. SJR indicator (2017): 0.339.

[TB17b] Tarasyuk I.V., Buchholz P. Behavioural equivalences for fluid stochastic Petri nets. CoRR abs/01706.2641 (arXiv:1706.02641), 54 pages, Computing Research Repository, Cornell University Library, Ithaca, NY, USA, June 2017, http://itar.iis.nsk.su/files/itar/pages/fspneqarxiv.pdf, http://arxiv.org/pdf/1706.02641.pdf.

*[BBTT17] Bause F., Buchholz P., Tarasyuk I.V., Telek M. Equivalence and lumpability of FSPNs. Proceedings of 24th International Conference on Analytical and Stochastic Modelling Techniques and Applications - 17 (ASMTA'17) (N. Thomas, M. Forshaw, eds.), Newcastle upon Tyne, UK, July 10-11, 2017, Lecture Notes in Computer Science 10378, pages 16-31, Springer, 2017 (ISSN 0302-9743, ISBN 978-3-319-61427-4), DOI: 10.1007/978-3-319-61428-1_2. Scopus, Springer indexed. SJR indicator (2016): 0.315.

*[TMV18a] Tarasyuk I.V., Macia S.H., Valero R.V. Bisimulation equivalence for functional and performance analysis of concurrent stochastically timed systems in dtsiPBC. Technical Report DIAB-18-05-1, 99

pages, Department of Computer Systems, High School of Computer Science Engineering, University of Castilla - La Mancha, Albacete, Spain, May 2018,
http://itar.iis.nsk.su/files/itar/pages/dtsipbctcsrwr.pdf,
http://www.dsi.uclm.es/descargas/technicalreports/DIAB-18-05-1/TR_DSI_may2018.pdf.

*[TMV18b] Tarasyuk I.V., Macia S.H., Valero R.V. Stochastic equivalence for performance analysis of concurrent systems in dtsiPBC. Siberian Electronic Mathematical Reports 15, pages 1743-1812, S.L. Sobolev Institute of Mathematics, Novosibirsk, December 2018 (ISSN 1813-3304),
DOI: 10.33048/semi.2018.15.144, http://itar.iis.nsk.su/files/itar/pages/dtsipbceqsemr.pdf,
http://semr.math.nsc.ru/v15/p1743-1812.pdf. Web of Science, Scopus, Zentralblatt Math indexed. SJR indicator (2018): 0.412.

[Tar19] Tarasyuk I.V. Discrete time stochastic and deterministic Petri box calculus. CoRR abs/1905.00456 (arXiv: 1905.00456), 57 pages, Computing Research Repository, Cornell University Library, Ithaca, NY, USA, May 2019, http://itar.iis.nsk.su/files/itar/pages/dtsdpbcarxiv.pdf,
http://arxiv.org/pdf/1905.00456.pdf.

*[TB19] Tarasyuk I.V., Buchholz P. Logical characterization of fluid equivalences. Siberian Electronic Mathematical Reports 16, pages 826-862, S.L. Sobolev Institute of Mathematics, Novosibirsk, June 2019 (ISSN 1813-3304), DOI: 10.33048/semi.2019.16.055,
http://itar.iis.nsk.su/files/itar/pages/fspneqlogsemr.pdf, http://semr.math.nsc.ru/v16/p826-862.pdf. Web of Science, Scopus, Zentralblatt Math indexed. SJR indicator (2018): 0.412. RSCI indexed. RSCI impact factor (2015): 0.265.

2.1. Project's initial questions and objectives.

A4. Developing and studying behavioral equivalences for fluid stochastic Petri nets (FSPNs).

(a) Constructing new fluid equivalences for FSPNs.

We planned to proceed with introducing and studying fluid stochastic bisimulations on FSPNs, to be able to reduce the size of FSPNs by applying quotienting while preserving their transient and stationary behaviors.

(b) Logical characterizations of the fluid equivalences.

We intended to give logical characterizations for the fluid stochastic bisimulations by fluid extensions of Probabilistic Modal Logic (PML) of K.G. Larsen and A. Skou [LS91] on probabilistic transitions systems and Continuous Stochastic Logic (CSL) of A. Aziz, K. Sanwal, V. Singhal and R. Brayton [ASSB00] on continuous time Markov chains. Our research was to rely on the work of G. Clark, S. Gilmore, J. Hillston and M. Ribaudo [CGHR00] about PML_μ, a stochastic extension of PML for PEPA, and that of M. Gribaudo and A. Horvath [GHo05] about a branching time temporal logic, an extension of CSL for stochastic fluid models, aiming to express their quantitative properties.

C1. Developing application examples for FSPNs analysis techniques.

(a) Case studies demonstrating the analysis techniques for FSPNs.

As new analysis techniques were planned to be developed for fluid stochastic Petri nets (FSPNs), we planned to investigate a number of case studies, with the goal of applying the techniques to more realistic time- and resource-dependent systems. We intended to build illustrative examples of FSPNs modeling computer, communication and manufacturing systems.

(b) Reducing the FSPN application models by the fluid equivalences.

We supposed to demonstrate how the fluid stochastic equivalences developed can be applied to reduce the state space of FSPNs (e.g., by quotienting their reachability graphs by fluid bisimulations instead of conventional stochastic ones) and thus to simplify both functional analysis and performance evaluation of the modeled example systems.

2.3. Presentation of results.

A4. Developing and studying behavioral equivalences for fluid stochastic Petri nets (FSPNs).

(a) Action labeling for transitions of FSPNs and their underlying stochastic/fluid models.

In [TB15,TB17a,TB17b,TB19], I.V. Tarasyuk and P. Buchholz (both project staff) have proposed fluid equivalences that allow one to compare and reduce behaviour of labeled fluid stochastic Petri nets (LFSPNs) while preserving their discrete and continuous properties. We have considered the LFSPNs whose continuous markings have no influence to the discrete ones, i.e. every discrete marking determines completely both the set of enabled transitions, their firing rates and the fluid flow rates of the incoming and outgoing arcs for each continuous place. Moreover, we have required that the discrete part of the LFSPNs should be continuous time stochastic Petri nets. The underlying stochastic model for the discrete part of the LFSPNs is continuous time Markov chains (CTMCs). The performance analysis of the continuous part of LFSPNs is accomplished via the associated stochastic fluid models (SFMs).

(b) Branching-time relation of fluid bisimulation equivalence for LFSPNs.

In [TB15,TB17a,TB17b,TB19], I.V. Tarasyuk and P. Buchholz (both project staff) have introduced a branching-time relation of fluid bisimulation equivalence. We have required the fluid bisimulation on the discrete markings of two LFSPNs to be a standard (strong) Markovian bisimulation. As for the continuous markings, we have required that, for every pair of the Markovian bisimilar discrete markings, the fluid flow rate of the continuous place in the first LFSPN should coincide with that of the continuous place in the second LFSPN. We have proven that fluid bisimulation equivalence preserves the following aggregated (by such a bisimulation) probability functions: stationary probability mass for the underlying CTMC, as well as stationary fluid buffer empty probability, fluid density and distribution for the associated SFM. Hence, the equivalence guarantees identity of a number of discrete and continuous performance measures. Fluid bisimulation equivalence has been then used to simplify the qualitative and quantitative analysis of LFSPNs that is accomplished by means of quotienting (by the equivalence) the discrete reachability graph and underlying CTMC. To describe the quotient associated SFM, the quotients of the probability functions have been defined.

(c) Linear-time relation of fluid trace equivalence for LFSPNs.

In [TB17a,TB17b,TB19], I.V. Tarasyuk and P. Buchholz (both project staff) have defined a linear-time relation of fluid trace equivalence. We have required that fluid trace equivalence on discrete markings of two LFSPNs should be a standard (strong) Markovian trace equivalence. Moreover, the average sojourn times in the respective discrete markings should be the same. Finally, for the two equivalent LFSPNs, the cumulative execution probabilities of all the paths corresponding to a particular sequence of actions, together with a concrete sequence of the average sojourn times, should be equal. As for the continuous markings of the two LFSPNs, we have further selected the paths with the same extracted action sequence and the same sequence of the extracted average sojourn times by counting the execution probabilities only of those paths additionally having the same sequence of extracted potential fluid flow rates of the respective continuous places. We have proven that fluid trace equivalence is strictly weaker than fluid bisimulation one. We have demonstrated that both the relations take into account the essential features of the LFSPNs behaviour: functional activity, stochastic timing and fluid flow. We have shown that fluid trace equivalence preserves average potential fluid change volume for the transition sequences of every certain length.

(d) Logical characterizations of fluid trace and bisimulation equivalences for LFSPNs.

In [TB17b,TB19], I.V. Tarasyuk and P. Buchholz (both project staff) have characterized logically fluid trace and bisimulation equivalences with two novel fluid modal logics HML_{flt} and HML_{flb}, constructed on the basis of the well-known Hennessy-Milner Logic HML. The logical characterizations guarantee that two LFSPNs are fluid (trace or bisimulation) equivalent iff they satisfy the same formulas of the respective fluid modal logic, i.e. they are logically equivalent. Thus, instead of comparing LFSPNs operationally, one may only check the corresponding satisfaction relation. This provides one with the possibility for logical reasoning on fluid equivalences for LFSPNs. These results can be also seen as operational characterizations of the corresponding logical equivalences. We have also explored how to adopt (if possible) the testing interpretations of probabilistic and Markovian equivalences (related to their logical characterizations) for fluid trace and bisimulation equivalences that are standardly defined in the operational manner. The fluid modal logic HML_{flt} is used to characterize fluid trace equivalence. Therefore, the interpretation function of the logic has an additional argument, which is the sequence of the potential fluid flow rates for the single continuous place of an LFSPN. The fluid modal logic HML_{flb} is intended to characterize fluid bisimulation equivalence. For this purpose, the logic has a new modality, decorated with the potential fluid flow rate value for the single continuous place of an LFSPN. We have demonstrated how the modalities and interpretation functions of the logics HML_{flt} and HML_{flb} respect the behavioural aspects of LFSPNs: semantics type (linear or branching time), functional activity (consisting in the action occurrences), stochastic timing (specified by the transition rates) and fluid flow (defined by the fluid rates).

(e) Stochastic process discretization and symmetries among continuous places towards general equivalences for FSPNs.

In [BBTT17], F. Bause, P. Buchholz, I.V. Tarasyuk (all project staff) and M. Telek (international partner) have considered on FSPNs more general equivalence relations that do not respect the action labels of transitions. Based on the equivalence relations for stochastic Petri nets (SPNs), which were derived from lumpability for Markov Chains, and from lumpability for certain classes of differential equations, we have defined an equivalence relation for FSPNs. Lumpability for the differential equations was based on a finite discretization approach and permutations of the fluid part of the FSPN. An appropriate equivalence relation allows one to aggregate the sets of equivalent states into single states. Such a state space reduction has been exploited for a more efficient analysis of FSPNs using a discretization approach. Lumpable equivalence relations have been computed as from an appropriately discretized state space of the stochastic process, as directly from the FSPN structure. Thus, we have extended the concept of lumping to FSPNs, which are hybrid (discrete- and continuous-state) systems. The fundamental approach behind this extension was to map the hybrid system into a discrete-state one and apply the available lumping relations for the discrete system. To pursue this approach, we have presented a discretization of FSPNs and elaborated on the refinement of the discretization step. We have shown that the refinement maintains the lumping relation, which is important for utilizing the fact that asymptotic behavior of the discrete system tends to the hybrid one as the discretization step tends to zero. We have also outlined an approach, where the lumping of the continuous part is based on the symmetry among continuous places, which is less general, but may be proved by generating only the discrete state space without building the discretized continuous part.

Relevant research situation.

In the literature, no transition labeling with actions or qualitative/quantitative behavioural equivalences for FSPNs have been proposed. In [ITV15], differential bisimulation (an ODE analogue of the stochastic one) for Fluid Extended Process Algebra (FEPA) has been constructed that induces a partition on ODEs of the FEPA terms. The aggregate ODE solution for each partition block is the sum of the solutions of its ODEs. In FSPNs, the ODE systems are obtained only in case of a single continuous place, since the FSPNs dynamics is generally described by the equation systems with partial derivatives w.r.t. the fluid levels in the continuous places. The levels are random variables with the FSPN work time parameter and the sojourn time in each discrete marking is a random variable as well. The FEPA processes are described by the ODE systems with time derivatives of the population functions that define the fluid atoms multiplicities, which are the functions of time and their values can be found at each time moment. The fluid levels in the continuous places of FSPNs are the continuous time-dependent random variables and their exact values at a given time cannot be calculated. The FEPA expressivity is restricted by parallel composition of fluid atoms, where no difference is made between "discrete" atoms with small multiplicities and "continuous" atoms with large ones. Thus, differential bisimulation cannot be transferred from FEPA to FSPNs. In [CTTV15], back and forth bisimulation equivalences on chemical species have been introduced for chemical reaction networks (CRNs) with the ODE-based semantics. Forth bisimulation induces a partition where each equivalence class is a sum of the species concentrations from it while back bisimulation relates the species with the same ODE solutions at all time points. Unlike CRNs, FSPNs have a stochastic behaviour, influenced by the time and probability interplay, and the FSPNs dynamics is analyzed with (multidimensional) SFMs, described by partial differential equations. In [CTTV16], back and forth differential equivalences (BDE and FDE) have been explored for Intermediate Drift Oriented Language (IDOL). BDE and FDE can be transferred from IDOL to the ODE-interpreted higher-level models, such as Petri nets, process algebras and rule systems. BDE and FDE embrace the lumpability-based minimization of CTMCs [DHS03], bisimulations of CRNs [CTTV15] and relations for process algebras with the ODE semantics [ITV15]. However, the IDOL-defined ODE class cannot specify stochastic continuous time delays in the discrete states of FSPNs. In [CTTV18], approximate (ε-) variants of BDE and FDE [CTTV16] have been defined within polynomial initial value problem (PIVP) over the set of ODE variables. Since ε-BDE and ε-FDE do not beware of actions, they are not behavioural relations. In [CTTV17a,TV18], syntactic Markovian bisimulation (SMB) has been proposed on CRNs with stochastic CTMC-based semantics. SMB is defined on the structure of CRNs rather than on their underlying CTMCs and is stricter than forth bisimulation [CTTV15]. SMB is not a traditional behavioural equivalence though: instead of action names, it takes species that specify the states of CRNs and their CTMCs. Forward and backward equivalences (FE and BE) have been considered in [CTTV17b] for polynomial ODE systems and in [TV18] for polynomial dynamical systems (PDSs). BE relates the ODE variables with the same solution while FE guarantees self-consistency of the ODE system for the sums of variables from the same equivalence classes. Since FE and BE do not respect actions, they are not standard behavioural relations. In [Bor17], L-bisimulation equivalence for the polynomial ODEs systems has been defined that agrees with the underlying ODEs. The equivalence does not take actions into account, hence, it is not a behavioural notion. The fluid equivalences constructed within the project respect both functional (action-based) and performance (stochastic timing- and fluid-based) properties of labeled FSPNs (LFSPNs).

In the literature, several logical characterizations of stochastic and Markovian equivalences have been proposed. In [CGHR00], the characterization of strong equivalence has been presented with the logic PML_μ, which is a stochastic extension of Probabilistic Modal Logic (PML) [LS91] on probabilistic transitions systems to the stochastic process algebra PEPA [Hil96]. In [GHo05], a branching time temporal logic has been described which is an extension of Continuous Stochastic Logic (CSL) [ASSB00] on CTMCs to a wide class of SFMs. The CSL-based logical characterizations of various stochastic bisimulation equivalences have been reported in [SD04,BKHW05,BHKP08] on labeled CTMCs, in [DP03] on labeled continuous time Markov processes (CTMPs), in [BHHHK13] on labeled Markov reward models (MRMs) and in [SZG14] on continuous time Markov decision processes (CTMDPs). In [Bern07], on sequential and concurrent Markovian process calculi SMPC (MPC) and CMPC, the logical characterizations of Markovian trace and bisimulation equivalences have been accomplished with the modal logics HML_{MTr} and HML_{MB}, based on Hennessy-Milner Logic (HML) [HM85]. In [BBo08], on (sequential) Markovian process calculus MPC, the logical characterizations of Markovian trace and bisimulation equivalences have been constructed with the HML-based modal logics HML_{NPMTr} and HML_{MB}. In the project, we have provided fluid trace and bisimulation equivalences with the logical characterizations, accomplished via formulas of the two novel fluid HML-based modal logics HML_{flt} and HML_{flb}.

The idea of lumping states in a discrete system has a long history in Markov chains [KS76,Buc94], but has also been used in linear systems [Cox84] and for differential equations [TLRT97]. Later it has been applied to specific modeling formalisms like stochastic process algebras [Hil96] and stochastic Petri nets [Buc95]. The current developments for fluid models can be found in [ITV15,TT14]. The central idea of lumpability is the definition of classes of states with an identical behavior and the substitution of the state classes by single states without altering the behavior of the system as it is observed. While our work on the project, we have implemented this approach for FSPNs in the following two versions. First, we have introduced a discretized version of the system and considered the discretized model lumping that involves identical behavior on the level of ordinary differential equations (ODEs). Second, we have explored the original FSPN lumping that presents identical behavior on the partial differential equations (PDEs) level.

Perspectives for application.

The results we have obtained for fluid stochastic models can be applied to optimize, simplify and make more effective construction, functional analysis and performance evaluation of many realistic systems, modeled by FSPNs. The examples of such systems are those with fluid flow (the movement of liquid or dry substances, transportation of energy amounts or information volumes), controlled by discrete logics, such as telecommunication, network, information, electrical, manufacturing, chemical and biological systems. The techniques we have developed can be further extended to a wider class of hybrid systems, i.e. those consisting of both discrete and continuous components that communicate by transmitting data types of different nature.

Follow-up research.

A possible continuation of the presented work may be characterization of the fluid equivalences via more expressive logics, such as the CSL [ASSB00] fluid extensions resembling the temporal logic for SFMs [GHo05]. In the future, we also plan to define a fluid place bisimulation relation that connects "similar" continuous places of LFSPNs, like place bisimulations [AS92,Tar98] relate discrete places of (standard) Petri nets. The lifting of the relation to the discrete-continuous LFSPN markings (with discrete markings treated as the multisets of places) will respect both the fluid distribution among the related continuous places and the rates of fluid flow through them. The summation of the fluid levels in the continuous places may be implemented with some constructions for extended FSPNs (EFSPNs) [Gri02]. EFSPNs have as deterministic fluid jump arcs, used to transfer a deterministic amount of fluid from one continuous place to another via intermediate stochastic transitions (deterministic fluid transfer), as random fluid jump arcs, used to transfer a random amount of fluid among continuous places (random fluid transfer). We can also use an enhancement of FSPNs with fluid transitions [Gri02] that transfer fluid from their input to output continuous places and implement fluid volume conservation. Next, we intend to apply to LFSPNs an analogue of the effective reduction technique from [AS92], based on place bisimulations, which allows one to merge the equivalent continuous places and transitions between them, thus resulting in the significant reductions in the LFSPNs structure. It would be also interesting to provide fluid place bisimulation equivalences with logical characterizations by constructing new fluid "place" logics, whose modalities are capable to specify "aggregate" fluid flow rates, i.e. the sums of the rates for the equivalent continuous places. At last, we plan to develop further our lumping approach to FSPNs, based as on discretization of the underlying stochastic process, as on symmetries among their continuous places.

Contribution of the German side.

As planned, the work on the equivalences for FSPNs has been undertaken by I.V. Tarasyuk (project staff) in cooperation with his German partner P. Buchholz (TU Dortmund, project staff), who has proposed to use Markovian bisimulation equivalences as a basis for defining behavioural equivalences for FSPNs. He has explained how to construct the quotients (by fluid bisimulation) of the transition rate matrix (TRM) for continuous time Markov chain (CTMC) and flow rate matrix (FRM) for the associated stochastic fluid model (SFM) of LFSPNs, using special collector and distributor matrices. The quotient TRMs and FRMs have been applied to describe the quotient associated SFMs of LFSPNs. P. Buchholz has also developed the methods of the stochastic process discretization and permutations of the fluid levels towards more general (transition labels independent) discrete-continuous equivalences on FSPNs.

C1. Developing application examples for FSPNs analysis techniques.

(a) Case study of the behavioural analysis via quotienting LFSPNs by fluid bisimulation equivalence.

In [TB17a,TB17b], I.V. Tarasyuk and P. Buchholz (both project staff) have constructed an application example of the document preparation system and in [TB17b,TB19], that of the plant production line, with a goal to demonstrate behavioural analysis via quotienting by fluid bisimulation equivalence, as well as logical specification of the qualitative and quantitative properties. The document preparation system receives (in an arbitrary order or in parallel) the collections of the text and graphics files as its inputs and writes them into the operative memory of a computer. The system then reads the (mixed) data from there and produces properly formatted output documents consisting of text and images. We have considered three different models of the system. Besides the two standard models (the first model with parallel and the second model with interleaving behaviour), the enhanced one (with interleaving behaviour) has been presented that makes difference between the low and high resolution graphics. We have calculated for the document preparation system a number of steady-state performance indices, based on the discrete and continuous measures of LFSPNs. The case study has also shown verification and comparison of the linear and branching-time behaviour with the new fluid modal logics HML_{flt} and HML_{flb}. For the plant production line with two liquid substances, mixed in a reservoir, we have verified formally the properties, specified as the probability given by the interpretation in HML_{flt} and the validity of the satisfaction in HML_{flb}.

(b) Case studies of lumping as the discrete process, as the matrices of continuous time Markov chains of FSPNs.

In [BBTT17], F. Bause, P. Buchholz, I.V. Tarasyuk (all project staff) and M. Telek (international partner) have demonstrated lumping the discrete process by example of the source and sink system with two non-symmetric fluid buffers. In this case study, summing the contents of the continuous places has been applied that resembles summing tokens in the equivalent discrete places when considering place bisimuation on standard Petri nets. Moreover, lumping the matrices of continuous time Markov chains (CTMCs) underlying the discrete part of the FSPNs has been considered in the example of the two-component switched producer and consumer system. While such a lumping, permutation of the fluid levels in continuous places has been demonstrated, which is typical for place bisimulations of Petri nets.

Relevant research situation.

In the literature, application examples have been considered only on unlabeled FSPNs, for which no behavioural equivalence has been defined. In [GHBTCM02], FSPNs have been applied as a model of the industrial temperature control system. In [BGH03], FSPNs have modeled the car security controller for road tunnels that informs about violation of minimal safe distance between vehicles, and several performance indices have been presented. In [GGMS05] FSPNS have been used to calculate the distributions of the transfer time in the peer-to-peer applications for the shared access to files. In the mentioned examples, various performance indices have been used. The performance measures that take into account only the discrete part of FSPNs (standardly, generalized SPNs, GSPNs), are called the discrete ones while the performance measures that (additionally) respect the continuous part of FSPNs (fluid flow) are named the continuous or the hybrid ones. Many discrete measures have been defined for different variants (continuous time, generalized, non-Markovian) of SPNs by M.K. Molloy, M.A. Marsan, G. Ciardo, J.K. Muppala, K.S. Trivedi, A. Bobbio, A. Puliafito, M. Telek, G. Conte, S. Donatelli, G. Franceschinis, G. Balbo et al. in 1982-2007. Some continuous measures have been introduced for (unlabeled) FSPNs by A. Bobbio, S. Garg, M. Gribaudo, A. Horvath, M. Sereno, M. Telek et al. in 1999-2002. It should be stressed that the number of the currently available continuous performance measures is much less than that of the discrete ones. Within the project, we have constructed a number of case studies on FSPNs, as the system models. Our goal was to show the advantages of applying behavioural equivalences to the FSPNs analysis that included calculating the performance indices, a wide range of which we have proposed, by taking special attention to the continuous/hybrid ones.

Perspectives for application.

The case studies we have presented demonstrate how to use in practice the theoretical results obtained within FSPNs and LFSPNs. The practical examples have shown which advantages and restrictions exist in the model of FSPNs, and which is the most appropriate application area for the theory proposed. Thus, the examples that we have constructed and investigated serve as a basis to develop more effective methods and powerful software tools, intended to analyze and evaluate behaviour of time-critical hybrid systems with stochastic timing and fluid flow.

Contribution of the German side.

The German partner P. Buchholz (TU Dortmund, project staff) in cooperation with I.V. Tarasyuk (project staff) has contributed into the work on the case studies with FSPNs by presenting an application example of the source and sink system to demonstrate summing the fluid levels of the continuous places of FSPNs. P. Buchholz has also constructed an example of the switched producer and consumer system that has shown lumping the matrices of the underlying continuous time Markov chains (CTMCs) of FSPNs and permutations of their continuous places.

***. Non-interleaving discrete time stochastic process algebras (SPAs).**

(a) Discrete time stochastic Petri box calculus with immediate multiactions (dtsiPBC) and its behavioural equivalences.

In [TMV14a,TMV14b,TMV15,TMV16,TMV17,TMV18a,TMV18b], I.V. Tarasyuk (project staff), H. Macia S. and V. Valero R. (both international partners) have proposed an extension with immediate multiactions of discrete time stochastic Petri box calculus (dtsPBC), presented in [Tar07] by I.V. Tarasyuk (project staff). The resulting algebra dtsiPBC is a discrete time analogue of stochastic Petri box calculus (sPBC) with immediate multiactions, introduced in [MVCR08] by H. Macia S., V. Valero R., F. Cuartero G. and M.C. Ruiz (all international partners) within a continuous time domain. In [MVCRT16], those four international partners and I.V. Tarasyuk (project staff) have presented the latest version of sPBC, applied to model the video conference system. In the latest version of dtsiPBC from [TMV17,TMV18a,TMV18b], we have used positive reals (instead of the previously used positive integers) as the weights of immediate multiactions to provide more flexibility in specification. The step operational semantics has been constructed via labeled probabilistic transition systems. The denotational semantics has been defined on the basis of a subclass of labeled discrete time stochastic Petri nets with immediate transitions. In order to evaluate performance, the corresponding semi-Markov chains (SMCs) and (reduced) discrete time Markov chains (DTMCs/RDTMCs) have been analyzed. We have defined the following behavioural algebraic equivalences: structural, w.r.t. transition systems and step stochastic bisimulation one. Then we have compared the differentiating power of the introduced relations. We have proven that the most strict relation, which is stochastic bisimulation equivalence, can be applied to reduce transition systems, underlying SMCs and associated DTMCs of expressions while preserving the functionality and performance characteristics. We have explained how this equivalence may help to simplify performance analysis of the algebraic processes.

(b) Case studies of performance analysis and reduction via quotienting by step stochastic bisimulation equivalence.

In [TMV16,TMV17,TMV18a,TMV18b], I.V. Tarasyuk (project staff), H. Macia S. and V. Valero R. (both international partners) have outlined in a case study a method of modeling, performance evaluation and behaviour preserving reduction (based on quotienting by the equivalence) of concurrent systems that allows one to simplify their analysis. The method has been applied to the two variants of the shared memory system: the standard one (all the multiaction probabilities and weights from the specification were fixed) and the generalized one (those probabilities and weights were interpreted as parameters to be adjusted). We have explored the effect of the parametric variation to the stationary probability mass function of the quotient SMC and to the corresponding performance measures of the generalized shared memory system. This allowed us to optimize the quantitative behaviour and achieve increase in performance of the system. We have also determined the main advantages of dtsiPBC by comparing it with other well-known or similar SPAs. In particular, by examining the interleaving transition system of the generalized shared memory system, we have demonstrated that step semantics is preferable to the interleaving one for the specification and analysis, as in our context, as within other discrete time SPAs.

(c) Discrete time stochastic Petri box calculus with deterministic multiactions (dtsdPBC).

In [Tar19], I.V. Tarasyuk (project staff) has proposed an extension with deterministically timed multiactions of discrete time stochastic and immediate Petri box calculus (dtsiPBC). In dtsdPBC, non-negative integers specify multiactions with fixed (including zero) time delays. The step operational semantics has been constructed via labeled probabilistic transition systems. The denotational semantics has been defined on the basis of a subclass of labeled discrete time stochastic Petri nets with deterministic transitions. The consistency of both semantics has been demonstrated. In order to evaluate performance, we have analyzed the corresponding semi-Markov chains with zero, one and geometrically distributed state residence times, as well as the respective (reduced) discrete time Markov chains.

Relevant research situation.

The semantics of all Markovian calculi known from the literature is interleaving, since their action delays have exponential distribution, which is the only continuous probability distribution with the memoryless (Markovian) property. Only a few non-interleaving stochastic process algebras have been proposed among non-Markovian ones [KA01]. However, none of such non-interleaving SPAs has the "explicitly" parallel operational semantics, since it has not been explained how to calculate the probability of the simultaneous execution of activities. In spite of the discrete time approach to constriction of some stochastic calculi, their operational semantics is still (decorated) interleaving. Concurrent execution is often interpreted as synchronization or the asynchronous completion of the activities which have already started. In the latter case of ST-operational semantics, the beginning of the activities execution, represented by the choice transitions, is accomplished in the interleaving way. Although several actions can end in parallel (within a step), the corresponding termination transitions have no labels. There exist only two discrete time SPAs: sACP [AHR00] and TCP^{dst} [MVi08]. In sACP and TCP^{dst}, delays are generally distributed, but a special attention is given to zero delays in sACP and to deterministic delays in TCP^{dst}. In sACP, immediate (timeless) transitions with zero delays serve as source and sink transitions of the dts-subnets corresponding to the choice, parallelism and iteration operators. In TCP^{dst}, zero delays of actions are specified by undelayable action prefixes while positive deterministic delays of processes are specified with timed delay prefixes. Neither formal syntax nor operational semantics for sACP were defined and it was not explained how to derive Markov chains from the algebraic expressions or the corresponding dts-nets to analyze performance. It was not stated explicitly, which type of semantics (interleaving or step) is accommodated in sACP. In spite of the discrete time approach, operational semantics of TCP^{dst} is still interleaving, unlike that of dtsiPBC. The new algebra dtsiPBC constructed within the project is a discrete time analog of stochastic Petri box calculus (sPBC) with immediate multiactions [MVCR08], proposed for continuous time domain. The salient point of dtsiPBC is a combination of immediate multiactions, discrete stochastic time and step semantics in a stochastic process algebra. The algebra dtsdPBC, being the extension of dtsiPBC with deterministically timed multiactions having a fixed time delay (including the zero one, which is the case of immediate multiactions) enhances the expressiveness of the latter and extends the application area of the associated analysis techniques.

Perspectives for application.

The results that we have obtained for dtsiPBC and dtsdPBC can be applied to functional and performance analysis of a wide variety of the discrete time-dependent concurrent and distributed systems, which can be naturally specified by the compositional algebraic structures, especially to the stochastically timed systems with mass parallelism. The behaviour of the systems is modeled with algebraic processes and can be compared by the algebraic stochastic equivalences. Hence, one can formally verify the systems and simplify them while preserving their qualitative and quantitative behaviour. The class of parallel systems to which our approach is well-suited includes computer, telecommunication systems and networks as well as many different kinds of manufacturing, information and service systems with concurrency, time constraints and stochasticity.

Follow-up research.

Future work will consist in constructing a congruence relation for dtsiPBC, i.e. the equivalence that withstands application of all operations of the algebra. One possible candidate is a stronger version of step stochastic bisimulation equivalence that is defined via the labeled probabilistic transition systems, equipped with two extra transitions, called "skip" and "redo". Moreover, recursion could be added to dtsiPBC to increase further specification power of the algebra. Further, in dtsdPBC, we plan to define step stochastic equivalences, including the bisimulation one, aiming to compare and reduce behaviour of the modeled systems, as well as to simplify their qualitative and quantitative analysis.

List of publications.

(a) Papers of the project participants (published during the DFG grant period).

[TMV14a] Tarasyuk I.V., Macia S.H., Valero R.V. Performance analysis of concurrent systems in algebra dtsiPBC. Programming 40(5), pages 3-27, MAIK Nauka / Interperiodica, Moscow, Russia, September 2014 (ISSN 0132-3474, in Russian), http://itar.iis.nsk.su/files/itar/pages/dtsialtpro.pdf.

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3.1. Presentation of the key scientific findings and any potential applications.

A4. Developing and studying behavioral equivalences for fluid stochastic Petri nets (FSPNs).

We have proposed fluid equivalences, to compare and reduce behaviour of labeled FSPNs (LFSPNs) while preserving their properties. We have defined a linear-time relation of fluid trace equivalence and its branching-time counterpart, fluid bisimulation one. Both fluid relations respect the essential features of the LFSPNs behaviour: functional activity, stochastic timing and fluid flow. The underlying stochastic model for the discrete part of the LFSPNs is continuous time Markov chains (CTMCs). The performance analysis of the continuous part of LFSPNs is accomplished via the associated stochastic fluid models (SFMs). We have shown that fluid trace equivalence preserves average potential fluid change volume for the transition sequences of every certain length. We have proven that fluid bisimulation equivalence preserves the aggregated (by the bisimulation) probability functions: stationary probability mass for the underlying CTMC, as well as stationary fluid buffer empty probability, fluid density and distribution for the associated SFM. Hence, the latter equivalence guarantees identity of the discrete and continuous performance measures. Fluid bisimulation equivalence has been used to simplify the qualitative and quantitative analysis of LFSPNs via quotienting the discrete reachability graph, underlying CTMC and associated SFM. We have characterized logically fluid trace and bisimulation equivalences with two novel fluid modal logics HML_{flt} and HML_{flb}.

We have considered equivalence relations for Fluid Stochastic Petri Nets (FSPNs). Based on equivalence relations for Stochastic Petri Nets (SPNs), which were derived from lumpability for Markov Chains, and from lumpability for certain classes of differential equations, we have defined an equivalence relation for FSPNs. Lumpability for the differential equations was based on a finite discretization approach and permutations of the fluid part of the FSPN. An appropriate equivalence relation allows one to aggregate sets of equivalent states into single states. Such a state space reduction has been exploited for a more efficient analysis of FSPNs using a discretization approach. Lumpable equivalence relations have been computed as from an appropriately discretized state space of the stochastic process, as directly from the FSPN.

C1. Developing application examples for FSPNs analysis techniques.

We have constructed an application examples of a document preparation system and a plant production line, to demonstrate the behavioural analysis via quotienting by fluid bisimulation equivalence and logical specification of the properties. Besides the two standard models of a document preparation system (the first with parallel and the second with interleaving behaviour), the enhanced one (with interleaving behaviour) has been presented that makes difference between the low and high resolution graphics. We have calculated for the document preparation system a number of steady-state performance indices, based on some standard discrete and continuous measures of LFSPNs. The case study has also shown verification and comparison of the linear and branching-time behaviour with the use of new fluid modal logics HML_{flt} and HML_{flb}. For the plant production line, we have verified the properties, specified as the interpretation probability in HML_{flt} and the satisfaction validity in HML_{flb}.

We have demonstrated lumping the discrete process by example of the source and sink system with two non-symmetric fluid buffers. In that case study, summing the contents of the continuous places has been applied that resembles summing tokens in the equivalent discrete places when considering place bisimuation on standard Petri nets. Moreover, lumping the matrices of continuous time Markov chains (CTMCs) underlying the discrete part of the FSPNs has been considered in the example of the two-component switched producer and consumer system. While such a lumping, permutation of the fluid levels in continuous places has been demonstrated, which is typical for place bisimulations of Petri nets.

***. Non-interleaving discrete time stochastic process algebras (SPAs).**

A new calculus dtsiPBC which is an extension of dtsPBC with immediate multiactions has been constructed. In that version of dtsiPBC, we have use positive reals (instead of the previously used positive integers) as the weights of immediate multiactions to provide more flexibility in specification. The step operational semantics was based on labeled probabilistic transition systems while the denotational semantics was based on labeled discrete time stochastic and immediate Petri nets. In order to evaluate performance, the corresponding semi-Markov chains and (reduced) discrete time Markov chains have been analyzed. We have defined step stochastic bisimulation equivalence of expressions and proven that it can be applied to reduce their transition systems and underlying semi-Markov chains while preserving the functionality and performance characteristics. We have explained how this equivalence may help to simplify performance analysis of the algebraic processes.

In a case study, a method of modeling, performance evaluation and behaviour preserving reduction of concurrent systems has been outlined and applied to the standard and generalized (with parameters) shared memory system. We have explored the effect of the parametric variation to the stationary probability mass function of the quotient semi-Markov chain and to the corresponding performance measures of the generalized shared memory system. We have determined the main advantages of dtsiPBC by comparing it with other well-known or similar SPAs.

We have proposed a new algebra dtsdPBC, an extension of dtsiPBC by adding deterministic multiactions with positive delays. The calculus dtsdPBC has a parallel step operational semantics, based on labeled probabilistic transition systems and a denotational semantics in terms of a subclass of labeled discrete time stochastic and deterministic Petri nets. A technique of performance evaluation in the framework of the calculus has been presented that explores the corresponding stochastic process, which is a semi-Markov chain. It has been proven that the underlying discrete time Markov chain or its reduction by eliminating vanishing states may alternatively and suitably be studied for that purpose. The theory presented has been illustrated with an extensive series of examples, among which is the travel system application example demonstrating performance analysis within dtsdPBC.

3.2. Any surprises encountered in the course of the project and in the results obtained.

***. Non-interleaving discrete time stochastic process algebras (SPAs).**

While constructing a case study within dtsiPBC, we have discovered that it is very interesting to investigate the generalized version of the shared memory system as well. The generalized system has been specified by the process expressions with variable multiaction probabilities and weights, interpreted as parameters of the performance indices. The resulting performance measure functions have been analyzed with a goal to optimize the generalized system's specification. We have determined at which values of the parameters the performance measure functions reach their maximums or minimums, which has allowed us to increase performance of the generalized system in a clear, obvious and purely analytical way, by proper adjustment of those parameters. We have also examined an interleaving variant of the transition system of the generalized system's expression and thereby demonstrated that step semantics is preferable to the interleaving one for the specification and analysis, as in our context, as within other discrete time SPAs.

Statistics: who has been working, from when to when; who visited whom, and for how long, and on which topics.

Research visits of Dr. Igor V. Tarasyuk (four one-month visits) during his work within CAVER (the whole project duration between 2014 and 2019):

(1) November 19 - December 19, 2014 - Guest Researcher, grants DFG BE 1267/14-1 and RFBR 14-01-91334, at Faculty of Computer Science, TU Dortmund (supervisor Prof. Dr. Peter Buchholz). Research topic: Transition labeling for FSPNs and fluid bisimulation equivalence on labeled fluid stochastic Petri nets (LFSPNs).

(2) October 1 - October 30, 2015 - Guest Researcher, grants DFG BE 1267/14-1 and RFBR 14-01-91334, at Faculty of Computer Science, TU Dortmund (supervisor Prof. Dr. Peter Buchholz). Research topic: Fluid bisimulation equivalence for quotienting the branching-time behaviour of LFSPNs while preserving the functionality and performance.

(3) October 2 - October 31, 2016 - Guest Researcher, grants DFG BE 1267/14-1 and RFBR 14-01-91334, at Faculty of Computer Science, TU Dortmund (supervisor Prof. Dr. Peter Buchholz). Research topic: Fluid trace equivalence for comparing the linear-time behaviour of LFSPNs and case studies of the functional and performance analysis using behaviour-preserving reduction by fluid bisimulation equivalence.

(4) April 1 - April 30, 2018 - Guest Researcher, grant DFG BE 1267/14-1, at Faculty of Computer Science, TU Dortmund (supervisor Prof. Dr. Peter Buchholz). Research topic: Characterizations of fluid trace and bisimulation equivalences of LFSPNs by new fluid modal logics being consistent with the linear- or branching-time semantics.

(5) June 2 - July 1, 2019 - Guest Researcher, grant DFG BE 1267/14-1, at Faculty of Computer Science, TU Dortmund (supervisor Prof. Dr. Peter Buchholz). Research topic: Logical characterizations of fluid equivalences and application examples on LFSPNs.