Report on research supported by a DAAD scholarship

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1 Preliminaries

1.1 Scholarship specification

German Academic Exchange Service (Deutscher Akademischer Austauschdienst), grant A/05/05334, "Re-invitations for Former Scholarship Holders" ("Wiedereinladungen für ehemalige Stipendiaten").

1.2 Period of research

The work reported has been done between June 1 and August 31, 2005.

1.3 Supervisor

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2 The research area and motivation

Stochastic Petri nets (SPNs) are a well-known model for quantitative analysis of discrete dynamic event systems proposed in [37, 38, 24]. Essentially, SPNs are a high level language for specification and performance analysis of concurrent systems. A stochastic process corresponding to this formal model is a Markov chain generated and analysed by well-developed algorithms and methods. Firing probabilities distributed along continuous or discrete time scale are associated with transitions of an SPN. Thus, there exist SPNs with continuous and discrete time. Markov chains of the corresponding types are associated with the SPNs. As a rule, for SPNs with continuous time (CTSPNs), exponential or phase distributions of transition probabilities are used. For SPNs with discrete time (DTSPNs), geometric or combinations of geometric distributions are usually used. Transitions of CTSPNs fire one by one at continuous time moments. Hence, the semantics of this model is interleaving one. In this semantics, parallel computations are modeled by all possible execution sequences of their components. Transitions of DTSPNs fire concurrently in steps at discrete time moments. Hence, this model has step semantics. In this semantics, parallel computations are modeled by sequences of concurrent occurrences (steps) of their components. In [17, 18], a labeling for transitions of CTSPNs with action names was proposed. Labeling allows SPNs to model processes with functionally similar components: the transitions corresponding to the similar components are labeled by the same action. Moreover, one can compare labeled SPNs by different behavioural equivalences, and this makes possible to check stochastic processes specified by labeled SPNs for functional similarity. Therefore, one can compare both functional and performance properties, and labeled SPNs turn into a formalism for quantitative and qualitative analysis.

Algebraic calculi hold a special place among formal models for specification of concurrent systems and analysis of their behavioral properties. In such process algebras (PAs), a system or a process is specified by an algebraic formula. A verification of the properties is accomplished at a syntactic level by means of well-developed systems of equivalences, axioms and inference rules. One of the first PAs was CCS (Calculus of Communicating Systems) [36]. Process algebras has been acknowledged to be very suitable formalism to operate with real time and stochastic systems as well. In the last years, stochastic extensions of PAs called stochastic process algebras (SPAs) became very popular as a modeling framework. SPAs do not just specify actions that can happen (qualitative features) as usual process algebras, but they associate some quantitative parameters with actions (quantitative characteristics). The papers [29, 16, 27, 22, 53, 11] propose a variety of SPAs. Process algebras allow one to specify processes in a compositional way via an expressive formal syntax. On the other hand, Petri nets provide one with an ability for visual representation of a process structure and execution. Hence, the relationship between SPNs and SPAs is of particular interest, since it allows to combine advantages of the both models. For this, a semantics of algebraic formulas in terms of Petri nets is usually defined. In the stochastic case, the Markov chain of the stochastic process specified by an SPA formula is built based on the state transition graph of the corresponding SPN.

As a rule, stochastic process calculi proposed in the literature are interleaving. As a semantic area, the interleaving formalism of transition systems is used. For example, an extension of CCS with probabilities and time called $TPCCS$ was defined in [26]. An enrichment of BPA with probabilistic choice, $prBPA$, as well as an extension of $prBPA$ with parallel composition operator named ACP_π^+ have been proposed in [1]. A standard way for probabilistic extension of process algebras into the calculi of probabilistic transition systems was described in [30]. The most famous SPAs proposed so far are $PEPA$ [27], $TIPP$ [29] and $EMPA$ [10]. It is worth to mention the stochastic process calculus PPA constructed in [51, 52] as well. Therefore, an investigation of a stochastic extension for more expressive and powerful algebraic calculi is very important. At present, the development of step or "true concurrent" (such that parallelism is considered as a causal independence) SPAs is in the very beginning. One can mention a concurrent SPA of finite processes $StAFP_0$ with step semantics proposed in [15]. At the same time, there still exists no algebra of infinite concurrent stochastic processes.

Petri box calculus (PBC) is a flexible and expressive process algebra based on calculi CCS [36] and AFP_0 [33]. PBC was proposed more than 10 years ago [3], and it was well explored since that time [2, 14, 19, 32, 34, 12, 13, 20, 21, 23, 28, 4, 5, 31, 6, 7, 8, 9]. It was intended to become a tool for description of a Petri net structure and relationships between nets. Its goal was to propose a compositional semantics for high level constructs of concurrent programming languages in terms of elementary Petri nets. Thus, PBC serves as a bridge between theory and applications. Formulas of PBC are combined not from single actions (including the invisible one) and variables only, as in CCS, but from multisets of actions called multiactions (*basic formulas*) as well. In contrast to CCS, concurrency and synchronization are different operations (*concurrent constructs*). Synchronization is defined as a unary multi-way stepwise operation based on communication of actions and their conjugates. The other fundamental operations are sequence and choice (*sequential constructs*). The calculus includes also restriction and relabeling (*abstraction constructs*). To specify infinite processes, refinement, recursion and iteration operations were added (*hierarchical constructs*). Thus, unlike CCS, algebra PBC has an additional iteration construction to specify infiniteness in the cases when finite Petri nets can be used as the semantic interpretation. For PBC, denotational semantics in terms of a subclass of Petri nets equipped with interface and considered up to isomorphism was proposed. This subclass is called Petri boxes. Calculus PBC has step operational semantics in terms of labeled transition systems based on structural operational semantics (SOS) rules. Pomset operational semantics of PBC was defined in [34] such that the partial order information was extracted from "decorated" step traces. In these step sequences, multiactions were annotated with an information on the relative position of the expression part they were derived from. Last years, several extensions of PBC were presented.

A stochastic extension of PBC called stochastic Petri box calculus ($sPBC$) was proposed in [48, 49, 50, 39, 44, 45, 46, 35]. In $sPBC$, multiactions have stochastic durations that follow negative exponential distribution. Each multiaction is instantaneous and equipped with a rate that is a parameter of the corresponding exponential distribution. The execution of a multiaction is possible only after the corresponding stochastic time delay. Just a finite part of PBC was used for the stochastic enrichment. This means that $sPBC$ has neither refinement or recursion or iteration operations. Denotational semantics was defined in terms of a subclass of labeled continuous time stochastic Petri nets (CTSPNs) called stochastic Petri boxes (s-boxes). Calculus $sPBC$ has interleaving operational semantics in terms of labeled transition systems. Note that we have interleaving behaviour here because of the fact that a simultaneous firing of any two transitions has zero probability in accordance to the properties of continuous time distributions. Current research in this branch has an aim to extend the specification abilities of $sPBC$ and to define an appropriate congruence relation over algebraic formulas. Recent results on constructing iteration for $sPBC$ were reported in [41, 42]. In the papers [40, 43], a number of new equivalence relations were proposed for regular terms of $sPBC$ to choose later a suitable candidate for a congruence. In [47] special multiactions with zero time delay were added to $sPBC$. Denotational semantics of such a $sPBC$ extension was defined via a subclass of labeled generalized SPNs (GSPNs). The subclass is called generalized stochastic Petri boxes (gs-boxes).

Nevertheless, there is still no stochastic extension of PBC with step semantics. It could be done with the use of labeled DTSPNs as a semantic area, since discrete time models allow for concurrent action occurrences. The enrichment based of DTSPNs would be more natural than interleaving one based on CTSPNs because PBC

has step denotational and operational semantics. Hence, it is worth to propose the discrete time stochastic enrichment of PBC based on DTSPNs.

3 The results obtained

We proposed a discrete time stochastic extension of finite PBC called $dtsPBC$. The work consisted of the following stages. First, we presented syntax of $dtsPBC$. Each multiaction of the initial calculus PBC was associated with a conditional probability. Such a pair is called stochastic multiaction or activity. Second, we proposed semantics of $dtsPBC$. Step operational semantics was constructed in terms of labeled transition systems based on action and inaction rules. The complexity here was a careful elaboration of step probabilities for formulas with parallelism and synchronization as well as the conflict resolving mechanism related to the probabilistic choice. Denotational semantics was defined in terms of a subclass of labeled DTSPNs (LDTSPNs) called discrete time stochastic Petri boxes (dts-boxes). At last, we defined a number of probabilistic equivalences in the algebraic setting based of transition systems without empty behaviour. These relations are weaker than the semantic equivalence of $dtsPBC$. They are used to identify stochastic processes with similar behaviour which are differentiated by the semantic equivalence that is too strict in many cases. Moreover, the proposed equivalences could be used to construct later a congruence relation based on one of them. In the best case, a complete and correct finite axiomatization of the congruence could be constructed. The hard task here would be to find a congruence that is not too distinctive, i.e., it should differentiate formulas with really different behaviour only in accordance to our needs. Moreover, the relation is to be axiomatizable and easy to check.

3.1 Syntax

Let $Act = \{a, b, \ldots\}$ be the set of *elementary actions*. Then $\widehat{Act} = \{\hat{a}, \hat{b}, \ldots\}$ be the set of *conjunctive actions (conjugates)* such that $a \neq \hat{a}$ and $\hat{\hat{a}} = a$. Let $\mathcal{A} = Act \cup \widehat{Act}$ be the set of *all actions*, and $\mathcal{L} = I\!N_f^{\mathcal{A}}$ be the set of *all multiactions*. The *alphabet* of $\alpha \in \mathcal{L}$ is defined as $\mathcal{A}(\alpha) = \{x \in \mathcal{A} \mid \alpha(x) > 0\}$.

An *activity (stochastic multiaction)* is a pair (α, ρ), where $\alpha \in \mathcal{L}$ and $\rho \in (0; 1)$ is the probability of multiaction α. Let $\mathcal{SL}$ be the set of *all activities*. The *alphabet* of $(\alpha, \rho) \in \mathcal{SL}$ is defined as $\mathcal{A}(\alpha, \rho) = \mathcal{A}(\alpha)$. For $(\alpha, \rho) \in \mathcal{SL}$, we define its *multiaction part* as $\mathcal{L}(\alpha, \rho) = \alpha$ and its *probability part* as $\Omega(\alpha, \rho) = \rho$.

Activities are combined into formulas by the following operations: *sequential execution* ;, *choice* [], *parallelism* $\|$, *relabeling* $[f]$, *synchronization* sy and *restriction* rs.

Relabeling functions $f : \mathcal{A} \to \mathcal{A}$ are bijections preserving conjugates, i.e., $\forall x \in \mathcal{A}\ f(\hat{x}) = \widehat{f(x)}$. Let $\alpha, \beta \in \mathcal{L}$ be two multiactions such that for some action $a \in Act$ we have $a \in \alpha$ and $\hat{a} \in \beta$ or $\hat{a} \in \alpha$ and $a \in \beta$. Then synchronization of α and β by a is defined as $\alpha \oplus_a \beta = \gamma$, where

$$\gamma(x) = \begin{cases} \alpha(x) + \beta(x) - 1, & \text{if } x = a \text{ or } x = \hat{a}; \\ \alpha(x) + \beta(x), & \text{otherwise.} \end{cases}$$

Definition 3.1 *Let $(\alpha, \rho) \in \mathcal{SL}$ and $a \in Act$. A* static expression *of $dtsPBC$ is defined as*

$$E ::= \ (\alpha, \rho) \mid E; E \mid E[]E \mid E\|E \mid E[f] \mid E \ \mathsf{rs}\ a \mid E \ \mathsf{sy}\ a.$$

Let $StatExpr$ denote the set of *all static expressions* of $dtsPBC$.

Definition 3.2 *Let $(\alpha, \rho) \in \mathcal{SL}$ and $a \in Act$. A* dynamic expression *of $dtsPBC$ is defined as*

$$G ::= \ \overline{E} \mid \underline{E} \mid G; E \mid E; G \mid G[]E \mid E[]G \mid G\|G \mid G[f] \mid G \ \mathsf{rs}\ a \mid G \ \mathsf{sy}\ a.$$

Let $DynExpr$ denote the set of *all dynamic expressions* of $dtsPBC$.

3.2 Operational semantics

We construct step operational semantics in terms of labeled transition systems.

3.2.1 Inaction rules

Let $E, F \in StatExpr,\ G \in DynExpr$ and $a \in Act$.

$$\begin{array}{lllll}
\overline{E;F} \xrightarrow{\emptyset} \overline{E};F & \underline{E};F \xrightarrow{\emptyset} E;\overline{F} & E;\underline{F} \xrightarrow{\emptyset} \underline{E;F} & \overline{E[]F} \xrightarrow{\emptyset} \overline{E}[]F & \overline{E[]F} \xrightarrow{\emptyset} E[]\overline{F} \\
\underline{E}[]F \xrightarrow{\emptyset} \underline{E[]F} & E[]\underline{F} \xrightarrow{\emptyset} \underline{E[]F} & \overline{E\|F} \xrightarrow{\emptyset} \overline{E}\|\overline{F} & \underline{E}\|\underline{F} \xrightarrow{\emptyset} \underline{E\|F} & \overline{E[f]} \xrightarrow{\emptyset} \overline{E}[f] \\
\underline{E}[f] \xrightarrow{\emptyset} \underline{E[f]} & \overline{E\ \mathsf{rs}\ a} \xrightarrow{\emptyset} \overline{E}\ \mathsf{rs}\ a & \underline{E}\ \mathsf{rs}\ a \xrightarrow{\emptyset} \underline{E\ \mathsf{rs}\ a} & \overline{E\ \mathsf{sy}\ a} \xrightarrow{\emptyset} \overline{E}\ \mathsf{sy}\ a & \underline{E}\ \mathsf{sy}\ a \xrightarrow{\emptyset} \underline{E\ \mathsf{sy}\ a} \\
G \xrightarrow{\emptyset} G & & & &
\end{array}$$

Let $E \in StatExpr,\ G, H, \widetilde{G}, \widetilde{H} \in DynExpr$ and $a \in Act$.

$$\frac{G \xrightarrow{\emptyset} \widetilde{G},\ \circ \in \{;, []\}}{G \circ E \xrightarrow{\emptyset} \widetilde{G} \circ E} \quad \frac{G \xrightarrow{\emptyset} \widetilde{G},\ \circ \in \{;, []\}}{E \circ G \xrightarrow{\emptyset} E \circ \widetilde{G}} \quad \frac{G \xrightarrow{\emptyset} \widetilde{G}}{G\|H \xrightarrow{\emptyset} \widetilde{G}\|H} \quad \frac{H \xrightarrow{\emptyset} \widetilde{H}}{G\|H \xrightarrow{\emptyset} \widetilde{G}\|H} \quad \frac{G \xrightarrow{\emptyset} \widetilde{G}}{G[f] \xrightarrow{\emptyset} \widetilde{G}[f]} \quad \frac{G \xrightarrow{\emptyset} \widetilde{G},\ \circ \in \{\mathsf{rs}, \mathsf{sy}\}}{G \circ a \xrightarrow{\emptyset} \widetilde{G} \circ a}$$

A dynamic expression G is *operative* if no inaction rule can be applied to it. Let $OpDynExpr$ denote the set of *all operative dynamic expressions* of $dtsPBC$.

Let $\simeq = (\xrightarrow{\emptyset} \cup \xleftarrow{\emptyset})^*$ be dynamic expression isomorphism in $dtsPBC$.

3.2.2 Action rules

Let $(\alpha, \rho), (\beta, \chi) \in \mathcal{SL},\ E \in StatExpr,\ G, H \in OpDynExpr,\ \widetilde{G}, \widetilde{H} \in DynExpr$ and $a \in Act$. Moreover, let $\Gamma, \Delta \in I\!N_f^{\mathcal{SL}}$. The *alphabet* of $\Gamma \in I\!N_f^{\mathcal{SL}}$ is defined as $\mathcal{A}(\Gamma) = \cup_{(\alpha,\rho)\in\Gamma}\mathcal{A}(\alpha)$.

$$\frac{}{\overline{(\alpha,\rho)} \xrightarrow{\{(\alpha,\rho)\}} \underline{(\alpha,\rho)}} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{G;E \xrightarrow{\Gamma} \widetilde{G};E} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{E;G \xrightarrow{\Gamma} E;\widetilde{G}} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{G[]E \xrightarrow{\Gamma} \widetilde{G}[]E} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{E[]G \xrightarrow{\Gamma} E[]\widetilde{G}} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{G\|H \xrightarrow{\Gamma} \widetilde{G}\|H} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{H\|G \xrightarrow{\Gamma} H\|\widetilde{G}}$$

$$\frac{G \xrightarrow{\Gamma} \widetilde{G},\ H \xrightarrow{\Delta} \widetilde{H}}{G\|H \xrightarrow{\Gamma+\Delta} \widetilde{G}\|H} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{G[f] \xrightarrow{f(\Gamma)} \widetilde{G}[f]} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G},\ a, \hat{a} \notin \mathcal{A}(\Gamma)}{G\ \mathsf{rs}\ a \xrightarrow{\Gamma} \widetilde{G}\ \mathsf{rs}\ a} \quad \frac{G \xrightarrow{\Gamma} \widetilde{G}}{G\ \mathsf{sy}\ a \xrightarrow{\Gamma} \widetilde{G}\ \mathsf{sy}\ a}$$

$$\frac{G\ \mathsf{sy}\ a \xrightarrow{\Gamma+\{(\alpha,\rho)\}+\{(\beta,\chi)\}} \widetilde{G}\ \mathsf{sy}\ a,\ a \in \mathcal{A}(\alpha),\ \hat{a} \in \mathcal{A}(\beta)}{G\ \mathsf{sy}\ a \xrightarrow{\Gamma+\{(\alpha\oplus_a\beta, \rho\cdot\chi)\}} \widetilde{G}\ \mathsf{sy}\ a}$$

3.2.3 Transition systems

For a dynamic expression G, we define the *transition system* $TS(G)$ and the *underlying discrete time Markov chain (DTMC)* $DTMC(G)$.

Example 3.1 *Let $E_1 = (\{a\}, \rho)[](\{a\}, \rho),\ E_2 = (\{b\}, \chi)$ and $E = E_1; E_2$. The identical activities of the composite static expression are enumerated as follows: $E = ((\{a\}, \rho)_1[](\{a\}, \rho)_2); (\{b\}, \chi)$. In Figure 1 the transition system $TS(\overline{E})$ and the underlying DTMC $DTMC(\overline{E})$ are presented. Note that for the reason of simplicity in the graphical representation states are depicted by expressions belonging to the corresponding equivalence classes, and singleton multisets are written without braces.*

3.3 Denotational semantics

We construct denotational semantics in terms of a subclass of labeled DTSPNs called discrete time stochastic Petri boxes (dts-boxes).

3.3.1 Labeled DTSPNs

We propose a class on *labeled DTSPNs (LDTSPNs)*, an extension of DTSPNs with transition labeling. For a LDTSPN N, we define the *reachability graph* $RG(N)$ and the *underlying discrete time Markov chain (DTMC)* $DTMC(N)$.

Example 3.2 *In Figure 2 an LDTSPN with two visible transitions t_1 (labeled by a), t_2 (labeled by b) and and one invisible transition t_3 (labeled by τ) is depicted. Transition probabilities of N are denoted by $\rho_i = \Omega_N(t_i)$ $(1 \leq i \leq 3)$. In the figure one can see the reachability graph $RG(N)$ and the underlying DTMC $DTMC(N)$ as well. The reachability set consists of markings $M_1 = (1,1,0),\ M_2 = (0,1,1),\ M_3 = (1,0,1),\ M_4 = (0,0,2)$.*

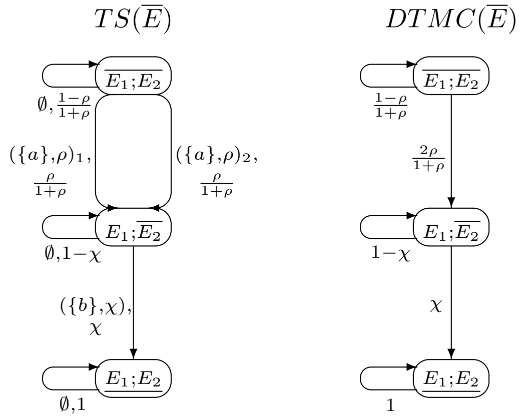

Figure 1: The transition system and the underlying DTMC of $\overline{E} = \overline{((\{a\},\rho)_1[](\{a\},\rho)_2);(\{b\},\chi)}$

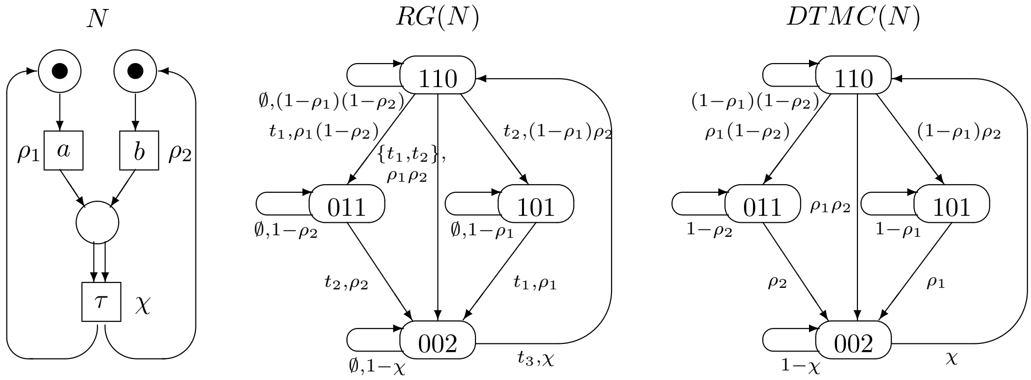

Figure 2: LDTSPN, its reachability graph and the underlying DTMC

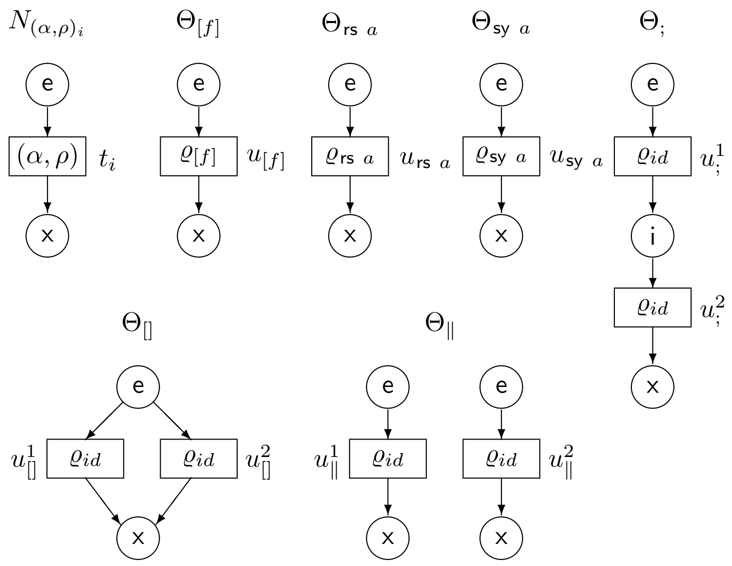

Figure 3: The plain and operator dts-boxes

3.3.2 Algebra of dts-boxes

We propose a class of *plain discrete time stochastic Petri boxes (plain dts-boxes)*, a discrete time stochastic extension of plain Petri boxes from PBC. The structure of the plain dts-box corresponding to a static expression is constructed as in PBC, i.e., via refinement and labeling. The plain and operator dts-boxes are presented in Figure 3.

Definition 3.3 *Let* $(\alpha,\rho) \in \mathcal{SL}$ *and* $E, F, \in StatExpr$. *The* denotational semantics $dtsPBC$ *is a mapping* Box_{dts} *from* $StatExpr$ *into the area of plain dts-boxes defined as follows:*

1. $Box_{dts}((\alpha,\rho)_i) = N_{(\alpha,\rho)_i}$;
2. $Box_{dts}(E \circ F) = \Theta_\circ(Box_{dts}(E), Box_{dts}(F)),\ \circ \in \{;, [], \|\}$;
3. $Box_{dts}(E[f]) = \Theta_{[f]}(Box_{dts}(E))$;
4. $Box_{dts}(E \circ a) = \Theta_{\circ a}(Box_{dts}(E)),\ \circ \in \{\mathsf{rs}, \mathsf{sy}\}$.

We denote isomorphism of transition systems by $\simeq$, and the same symbol denotes isomorphism of reachability graphs and DTMCs. Moreover, $\simeq$ will denote an isomorphism between transition systems and reachability graphs.

Theorem 3.1 *For any static expression* E

$$TS(\overline{E}) \simeq RG(Box_{dts}(E), {}^\circ Box_{dts}(E)).$$

Definition 3.4 *Two dynamic expressions* G *and* G' *are* equivalent w.r.t. semantics of $dtsPBC$, *denoted by* $G =_{dts} G'$, *if* $TS(G) \simeq TS(G')$.

Proposition 3.1 *For any static expression* E

$$DTMC(\overline{E}) \simeq DTMC(Box_{dts}(E), {}^\circ Box_{dts}(E)).$$

Example 3.3 *Let* $E_1 = (\{a\},\rho)$, $E_2 = (\{\hat{a}\},\chi)$ *and* $E = (E_1\|E_2)$ sy $a = ((\{a\},\rho)\|(\{\hat{a}\},\chi))$ sy a. *In Figure 4 the transition system* $TS(\overline{E})$ *and the underlying DTMC* $DTMC(\overline{E})$ *are presented. In Figure 5 the marked dts-box* $N = (Box_{dts}(E), {}^\circ Box_{dts}(E))$, *its reachability graph* $RG(N)$ *and the underlying DTMC* $DTMC(N)$ *are presented. It is easy to see that* $TS(\overline{E})$ *and* $RG(N)$ *are isomorphic as well as* $DTMC(\overline{E})$ *and* $DTMC(N)$.

The probabilities $\mathcal{P}_{ij}$ $(1 \le i, j \le 4)$ *are calculated as follows. Note that the symbol* sy *inscribes probability of the transition generated by synchronization, and the symbol* $\|$ *inscribes that of the transition corresponding*

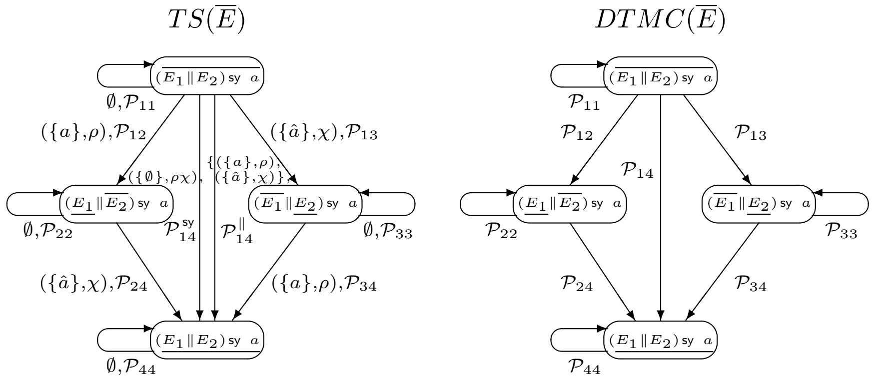

Figure 4: The transition system and the underlying DTMC of $\overline{E} = \overline{((\{a\},\rho)\|(\{\hat{a}\},\chi))\ \mathsf{sy}\ a}$

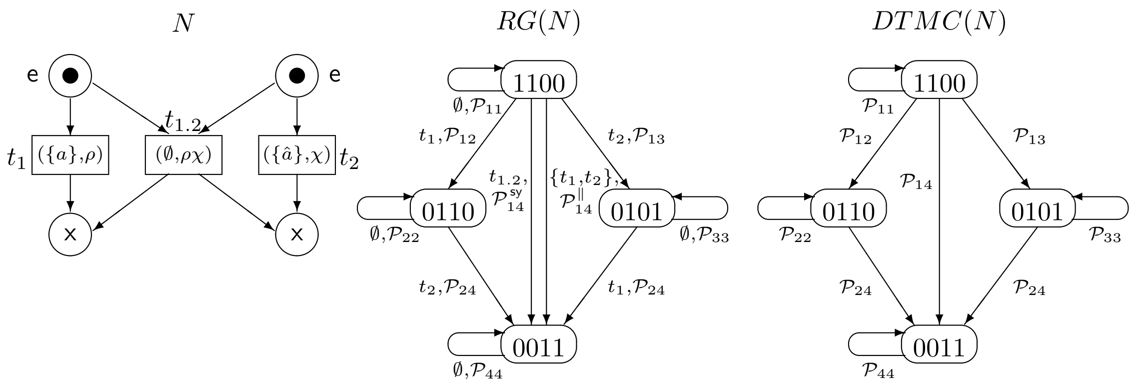

Figure 5: The marked dts-box $N = (Box_{dts}(E), {}^{\circ}Box_{dts}(E))$, its reachability graph and the underlying DTMC

to the concurrent execution of two activities. To avoid complex notation, we use the normalization factor $\mathcal{N} = \frac{1}{1-\rho^2\chi-\rho\chi^2+\rho^2\chi^2}$.

$$\begin{array}{lll}
\mathcal{P}_{11} = \mathcal{N}(1-\rho)(1-\chi)(1-\rho\chi) & \mathcal{P}_{12} = \mathcal{N}\rho(1-\chi)(1-\rho\chi) & \mathcal{P}_{13} = \mathcal{N}\chi(1-\rho)(1-\rho\chi) \\
\mathcal{P}^{\mathsf{sy}}_{14} = \mathcal{N}\rho\chi(1-\rho)(1-\chi) & \mathcal{P}^{\|}_{14} = \mathcal{N}\rho\chi(1-\rho\chi) & \mathcal{P}_{22} = 1-\chi \\
\mathcal{P}_{24} = \chi & \mathcal{P}_{33} = 1-\rho & \mathcal{P}_{34} = \rho \\
\mathcal{P}_{44} = 1 & \mathcal{P}_{14} = \mathcal{P}^{\mathsf{sy}}_{14} + \mathcal{P}^{\|}_{14} = \mathcal{N}\rho\chi(2-\rho-\chi) &
\end{array}$$

Consider the case $\rho = \chi = \frac{1}{2}$. *Then the transition probabilities will be the following:*

$$\mathcal{P}_{11} = \mathcal{P}_{12} = \mathcal{P}_{13} = \mathcal{P}^{\|}_{14} = \frac{3}{13},\ \mathcal{P}^{\mathsf{sy}}_{14} = \frac{1}{13},\ \mathcal{P}_{22} = \mathcal{P}_{24} = \mathcal{P}_{33} = \mathcal{P}_{34} = \frac{1}{2},\ \mathcal{P}_{44} = 1,\ \mathcal{P}_{14} = \frac{4}{13}.$$

3.4 Probabilistic equivalences

We propose a number of probabilistic equivalences of expressions. Semantic equivalence $=_{dts}$ is too strict in many cases, hence, we need weaker equivalence notions to compare behaviour of processes specified by algebraic formulas.

To identify processes with intuitively similar behavior, and to be able to apply standard constructions and techniques, we should abstract from infinite behaviour. Since $dtsPBC$ is a stochastic extension of finite PBC, the only source of infinite behaviour are empty loops, i.e., the transitions which do not change states and have empty multiaction parts of their labels. During such the abstraction, we should collect the probabilities of the empty loops. Note that the resulting probabilities are those defined for infinite number of empty steps.

For a dynamic expression G, we define the *transition system without empty loops* $TS^*(G)$ and the *underlying DTMC without empty loops* $DTMC^*(G)$. For a LDTSPN N, we define the *reachability graph without empty loops* $RG^*(N)$ and the *underlying DTMC without empty loops* $DTMC^*(N)$.

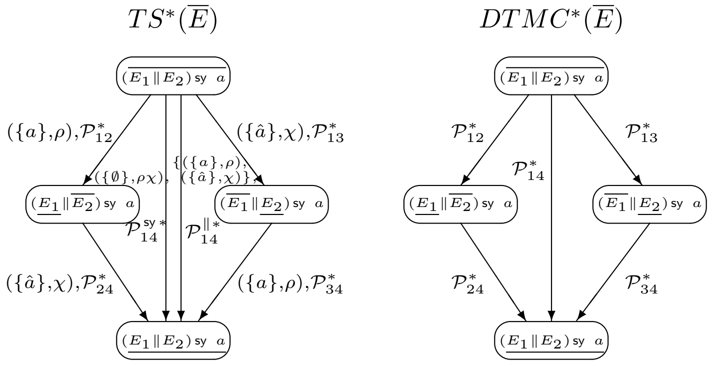

Figure 6: The transition system and the underlying DTMC without empty loops of $\overline{E}$ from Example 3.3

Theorem 3.2 *For any static expression E*

$$TS^*(\overline{E}) \simeq RG^*(Box_{dts}(E), {}^\circ Box_{dts}(E)).$$

Definition 3.5 *Two dynamic expressions G and G′ are* equivalent w.r.t. semantics of *dtsPBC* without empty loops*, denoted by* $G =_{dts*} G'$*, if* $TS^*(G) \simeq TS^*(G')$.

Proposition 3.2 *For any static expression E*

$$DTMC^*(\overline{E}) \simeq DTMC^*(Box_{dts}(E), {}^\circ Box_{dts}(E)).$$

Note that Theorem 3.2 guarantees that the net versions of algebraic equivalences could be easily defined.

Example 3.4 *Let E and N be those from Example 3.3. In Figure 6 the transition system* $TS^*(\overline{E})$ *and the underlying DTMC* $DTMC^*(\overline{E})$ *without empty loops are presented. In Figure 7 the reachability graph* $RG^*(N)$ *and the underlying DTMC* $DTMC^*(N)$ *without from empty loops are presented. It is easy to see that* $TS^*(\overline{E})$ *and* $RG^*(N)$ *are isomorphic as well as* $DTMC^*(\overline{E})$ *and* $DTMC^*(N)$*.*

The probabilities $\mathcal{P}^*_{ij}$ $(1 \le i,j \le 4)$ *are calculated as follows. Note that the symbol* sy *inscribes probability of the transition generated by synchronization, and the symbol* $\|$ *inscribes that of the transition corresponding to the concurrent execution of two activities. To avoid complex notation, we use the normalization factor* $\mathcal{N}^* = \frac{1}{\rho+\chi-2\rho^2\chi-2\rho\chi^2+2\rho^2\chi^2}$*. Note that the probabilities* $\mathcal{P}_{ij}$ $(1 \le i,j \le 4)$ *are taken from Example 3.3.*

$$\mathcal{P}^*_{12} = \frac{\mathcal{P}_{12}}{1-\mathcal{P}_{11}} = \mathcal{N}^*\rho(1-\chi)(1-\rho\chi) \qquad \mathcal{P}^*_{13} = \frac{\mathcal{P}_{13}}{1-\mathcal{P}_{11}} = \mathcal{N}^*\chi(1-\rho)(1-\rho\chi)$$
$$\mathcal{P}^{\mathsf{sy}\,*}_{14} = \frac{\mathcal{P}^{\mathsf{sy}}_{14}}{1-\mathcal{P}_{11}} = \mathcal{N}^*\rho\chi(1-\rho)(1-\chi) \qquad \mathcal{P}^{\|*}_{14} = \frac{\mathcal{P}^{\|}_{14}}{1-\mathcal{P}_{11}} = \mathcal{N}^*\rho\chi(1-\rho\chi)$$
$$\mathcal{P}^*_{24} = \frac{\mathcal{P}_{24}}{1-\mathcal{P}_{22}} = 1 \qquad \mathcal{P}^*_{34} = \frac{\mathcal{P}_{34}}{1-\mathcal{P}_{33}} = 1$$
$$\mathcal{P}^*_{14} = \mathcal{P}^{\mathsf{sy}\,*}_{14} + \mathcal{P}^{\|*}_{14} = \frac{\mathcal{P}^{\mathsf{sy}}_{14}+\mathcal{P}^{\|}_{14}}{1-\mathcal{P}_{11}} = \mathcal{N}^*\rho\chi(2-\rho-\chi)$$

Consider the case $\rho = \chi = \frac{1}{2}$*. Then the transition probabilities will be the following:*

$$\mathcal{P}^*_{12} = \mathcal{P}^*_{13} = \mathcal{P}^{\|*}_{14} = \frac{3}{10},\ \mathcal{P}^{\mathsf{sy}\,*}_{14} = \frac{1}{10},\ \mathcal{P}^*_{24} = \mathcal{P}^*_{34} = 1,\ \mathcal{P}^*_{14} = \frac{2}{5}.$$

Trace equivalences are the least distinctive ones. In the trace semantics, behavior of a system is associated with the set of all possible sequences of activities, i.e., protocols of work or computations. Thus, the points of choice of an external observer between several extensions of a particular computation are not taken into account. We defined interleaving ($\equiv_{ip}$) and step ($\equiv_{sp}$) probabilistic trace equivalences.

Bisimulation equivalences respect completely the particular points of choice in the behavior of a modeled system. We intend to present a parameterized definition of probabilistic bisimulation equivalences. We defined interleaving ($\underline{\leftrightarrow}_{ip}$) and step ($\underline{\leftrightarrow}_{sp}$) probabilistic bisimulation equivalences.

Stochastic isomorphism ($=_{sto}$) is a relation that is weaker than the equivalence with respect to the isomorphism of the associated transition systems without empty loops. The main idea is to summarize probabilities of all transitions between the same pair of states such that the transition labels have the same multiaction parts.

In the following, the symbol '_' will denote "nothing", and the equivalences subscribed by it are considered as those without any subscription.

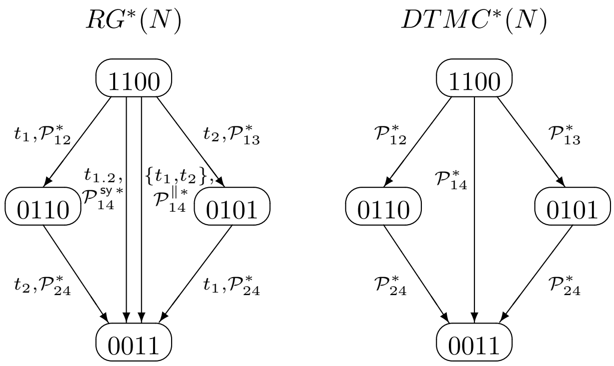

Figure 7: The reachability graph and the underlying DTMC without empty loops of N from Example 3.3

$$
\begin{tikzcd}
 & \simeq \arrow[d] \\
 & =_{dts} \arrow[d] \\
 & =_{dts*} \arrow[d] \\
 & =_{sto} \arrow[d] \\
\underline{\leftrightarrow}_{ip} \arrow[d] & \underline{\leftrightarrow}_{sp} \arrow[l] \arrow[d] \\
\equiv_{ip} & \equiv_{sp} \arrow[l]
\end{tikzcd}
$$

Figure 8: Interrelations of the probabilistic equivalences

Theorem 3.3 *Let* $\leftrightarrow, \twoheadleftarrow\!\!\!\twoheadrightarrow \in \{\equiv, \underline{\leftrightarrow}, =, \simeq\}$ *and* $\star, \star\star \in \{\_, ip, sp, sto, dts*, dts\}$*. For dynamic expressions* G *and* G'

$$G \leftrightarrow_\star G' \;\Rightarrow\; G \twoheadleftarrow\!\!\!\twoheadrightarrow_{\star\star} G'$$

iff in the graph in Figure 8 there exists a directed path from $\leftrightarrow_\star$ *to* $\twoheadleftarrow\!\!\!\twoheadrightarrow_{\star\star}$*.*

Example 3.5 *In Figure 9 the marked dts-boxes corresponding to the dynamic expressions from equivalence examples of Theorem 3.3 are presented, i.e.,* $\overline{E} = G$, $N = (Box_{dts}(E), {}^\circ Box_{dts}(E))$ *and* $\overline{E'} = G'$, $N' = (Box_{dts}(E'), {}^\circ Box_{dts}(E'))$ *for each picture (a)–(f). Since all the equivalences of dynamic expressions can be transferred to the corresponding marked dts-boxes, we depict also which the net analogues (denoted by the same symbols) of the algebraic equivalences relate the nets.*

4 The papers prepared

1. Tarasyuk I.V. *Discrete time stochastic Petri box calculus.* 25 p., 2005 (to be published).

5 Participating scientific events

- *Research Seminar on Stochastic Models*, Real Time and Concurrent Systems group (RTCS, `http://www.info-ab.uclm.es/fmc/`), University of Albacete (`http://www.info-ab.uclm.es`), Albacete, Spain, July 6–7, 2005. My talk there: "Labeled DTSPNs as a semantic area for stochastic process algebras" (`http://www.iis.nsk.su/persons/itar/lectuclm.pdf`).
- *Seminar on Dependability Engineering - 05 (SDE'05)*, Graduiertenkolleg TrustSoft (`http://trustsoft.uni-oldenburg.de`), CvO UO, Oldenburg, Germany, July 21, 2005, `http://se.informatik.uni-oldenburg.de/lehre/sose2005/seminar-programme/`. My *invited* talk there: "Equivalences for net models of concurrent stochastic systems" (`http://www.iis.nsk.su/persons/itar/lectruste.pdf`).

6 Teaching activity

- Delivering the lecture "Discrete time stochastic Petri nets: a model for analysis of stochastic concurrent systems" (`http://www.iis.nsk.su/persons/itar/lecoffis.pdf`), Oldenburger Forschungs- und Entwicklungsinstitut für Informatik-Werkzeuge und -Systeme (OFFIS, `http://www.offis.de`), Oldenburg, Germany.
- Giving the talk "Behavioural equivalences for stochastic models of concurrent systems", Weekly Internal Seminar of Parallel Systems Group, Department of Computer Science, Faculty II, CvO UO, Oldenburg, Germany.

7 Participating regular research meetings

- *Monthly Guest Colloquium*, OFFIS, Oldenburg, Germany.
- *Weekly PhD Students Seminar of Graduate School "TrustSoft"*, Department of Computer Science, Faculty II, CvO UO, Oldenburg, Germany.
- *Weekly Research Seminar of Project Group "P-Umlaut"*, Department of Computer Science, Faculty II, CvO UO, Oldenburg, Germany.
- *Monthly Internal Colloquium*, OFFIS, Oldenburg, Germany.
- *Weekly Internal Seminar of Parallel Systems Group*, Department of Computer Science, Faculty II, CvO UO, Oldenburg, Germany.
- *Weekly Research Seminar of Graduate and Postgraduate Students ("Diplomanden- und Doktorandenseminar (D+D)"*, Department of Computer Science, Faculty II, CvO UO, Oldenburg, Germany.

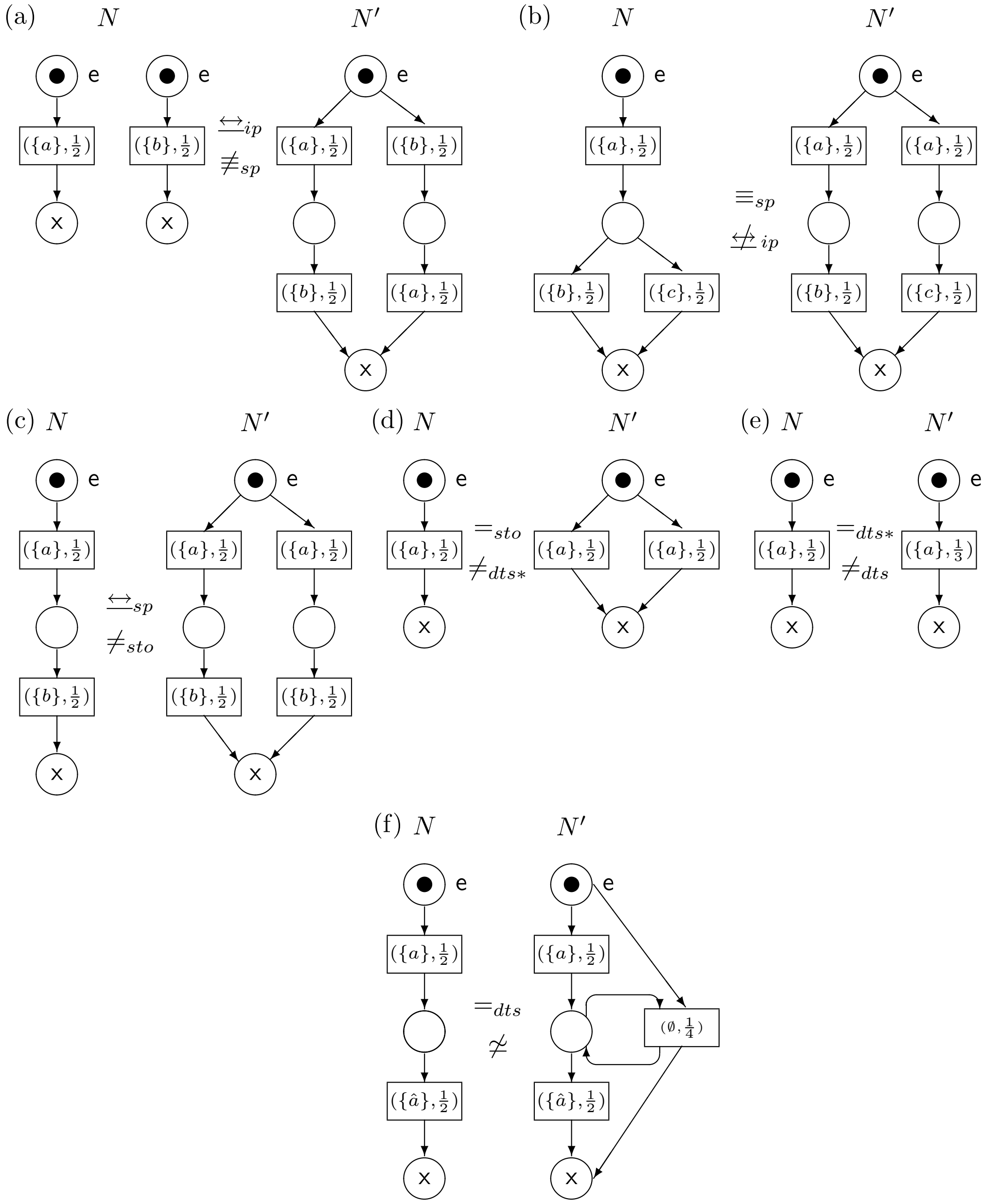

Figure 9: Dts-boxes of the dynamic expressions from equivalence examples of Theorem 3.3

8 Future research

Future work consists in the construction a congruence relation based on some probabilistic algebraic equivalence we defined. We can also abstract from the silent activities, i.e., those with empty multiaction part in the definitions of the equivalences. The abstraction from empty loops and that from silent activities could be done in one step as well. The main point is that we should collect probabilities during such the abstractions from the internal activity. As a result, we shall have the algebraic analogues of the net probabilistic equivalences from [15]. Moreover, we plan to extend *dtsPBC* with infiniteness constructs such as iteration and recursion. This research is planned to be done in close cooperation with the groups from Oldenburg, Germany, and Albacete, Spain.

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