Equivalences for Concurrent and Distributed Systems

A Survey of Dissertation [1]

Igor V. Tarasyuk

Theoretical Programming Laboratory,
A.P. Ershov Institute of Informatics Systems,
Siberian Division of the Russian Academy of Sciences,
Acad. Lavrentiev ave. 6, 630 090 Novosibirsk, Russia
Phone: +7 3832 34 13 60
Fax: +7 3832 32 34 94
E-mail: `itar@iis.nsk.su`

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Introduction

Importance of the problem considered. Last years, computing machines and complexes with parallel and distributed architecture became widespread, since they provide a possibility to solve a permanently growing volume of computation problems. But the problem of behavioral analysis for such concurrent systems appeared to be more complex than that for usual sequential systems, because some components of the former can work with partial of complete independence each of another. Therefore, such a branch of computer science as theory of parallel systems and processes becomes more and more important. It deals with an investigation of behavior of concurrent systems with the use of different mathematical formalisms.

In concurrency theory, from the very beginning, a great attention was to a development of formal models for specification and analysis of systems with independent occurrence of actions. In addition to such standard models as languages, automata and transition systems [179, 234], also that like Petri nets [228], process algebras [194, 200, 161, 162], Hoare traces [168], Mazurkiewicz traces [190, 191], synchronization trees [291] and event structures [211, 289, 292, 293] have been introduced.

In the papers [250, 251], a classification and comparison of the basic models is considered, which is in accordance with the three following dichotomies:

1. structure / behavior;

2. interleaving / true concurrency;

3. linear time / branching time.

In structure models, the states of systems are clearly described, and behavioral aspects are hidden. In behavior models, the actions fulfilled by systems are explicit ones, and the states can be derived from behavior.

In interleaving models, concurrency of actions is simulated by their nondeterministic interleaving (i.e occurrence in all possible orders). In true concurrency models, it is interpreted as causal independence of actions.

In linear time models, conflict is not respected, i.e. the moment of behavior when nondeterministic choice among several ways of further computations ("futures" or "branches") happens. In branching time models, this information is completely represented.

Changing a particular model corresponds to the choice of an abstraction level of behavior, i.e. to the changing of semantics.

An alternative (and more convenient) approach consists in the choice of an expressive enough (structural or behavioral) formal model and consideration of different equivalences which provide one with a semantics or an equality criteria for a proper abstraction level corresponding to the two remaining dichotomies.

Among these formalisms, a structural model of Petri nets and a behavioral model of process algebras became most popular. As mentioned in [275], the main advantages of Petri nets consist

in their ability for clear description of concurrency and long experience in both specification and analysis of parallel systems. In addition, the have useful graph representation. The main advantages of process algebras are in their modular nature, well developed equivalence notions, algebraic rules and complete proof systems. A considering these models together (in particular, a definition of a net semantics for algebraic formulas) provides one with a possibility to combine the best properties of both the formalisms.

A notion of equivalence is central in any theory of systems. Behavioral equivalences allow to compare systems taking into account particular aspects of their behavior and abstract from superfluous information. Equivalence relations are also used for behavior preserving reduction od systems and during verification, when the expected or high-level behavior of a system (specification) is compared with real or low-level one (implementation).

By present time, a lot of equivalences have been proposed. The most famous is the notion of bisimulation relation. Its great importance both for comparing and reduction of concurrent systems, proof of their correctness is mentioned many times in the literature.

The main known semantics defined by equivalences can be depicted by dots on the coordinate plane in Figure 1.

When moving along X axis, simulation of causality (or concurrency) grows: from interleaving semantics to true concurrent ones.

Moving along Y axis increases modeling of nondeterminism (conflict), i.e branching structure: from linear time semantics (trace) to branching time ones.

The following points on the X-axis are known.

Interleaving semantics A process (i.e. a possible computation of a system) is associated with a sequence of actions in the order of their occurrence.

Step semantics A process is associated with a sequence of multisets of actions.

Partial word semantics A process is associated with a partially ordered multiset of actions (pomset) with ordering by causal dependence. In this semantics, one system can simulate another if the ordering in it's pomset is less strict or the same as in the pomset of the second system. Thus, a pomset may be simulated by a less sequential (or a more parallel) one.

Pomset semantics A process is associated with a pomset, and the causal dependence relation should be preserved during simulation.

Process-net semantics A process is associated with an acyclic conflict-free net (called causal net), and a simulation requires isomorphism of causal nets. For convenience, we shall call the equivalences in this semantics just "process" ones.

The following points on the Y-axis are known.

Trace semantics The behavior of a system is completely defined by the set if it's processes. Two systems are considered to be equal in this semantics, if the sets of their processes coincide.

Bisimulation semantics The branching structure of the behavior of a system is taken into account, i.e. the points of choice between several alternative extensions of a present process are respected.

4

nondeter-
minism Y

isomorphism ●

conflict
preserving ●

history
preserving ●
bisimulation

ST-
bisimulation ●

usual ●
bisimulation

EQUIVALENCES

trace ●———●———●———●———●———→ X

interleaving   step   partial   pomset   process
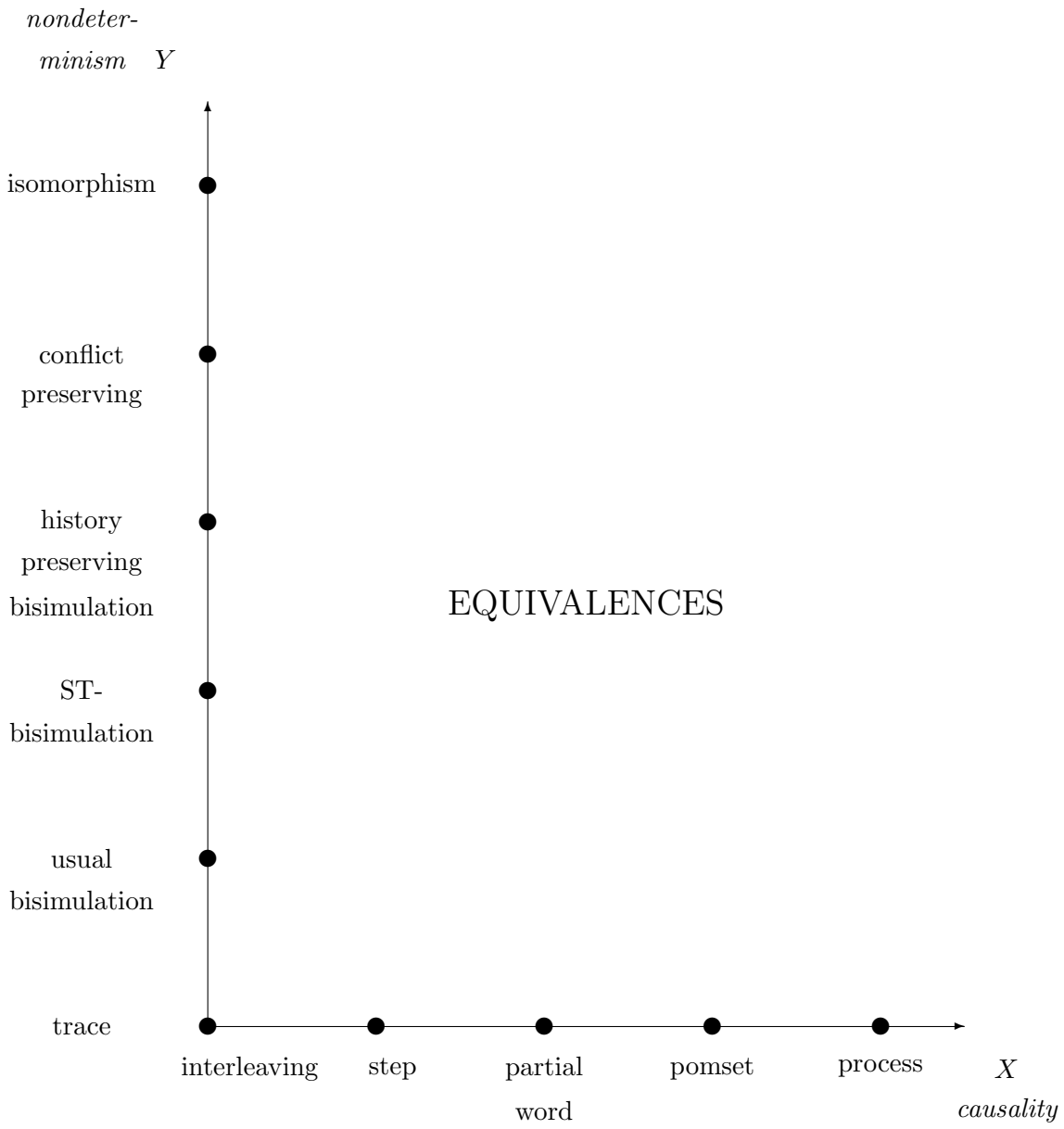
word   *causality*

Figure 1: Semantic coordinate plane

ST-bisimulation semantics Actions are considered as having some internal structure (or they occur not instantaneously, but for some period of time). During simulation, the causal dependence of both the actions of an extending process and that of the working actions of the present process are respected.

History preserving semantics This semantics takes into account the "past" of processes, i.e. the way how an extending process is causally connected with the present one.

Conflict preserving semantics The information about all the conflicts is respected.

Isomorphism This is the most strict semantics. Two systems are isomorphic if they differ only by names of their components.

In this branch of computer science, the following problems are open, and they are at present time a subject of a deep research.

- By now, there is no complete enough set of equivalences in all the considered semantics. Several candidates to fill the empty places on the coordinate plane in Figure 1 have been defined within different models, and this implies difficulty of their comparing. As mentioned in[159], for the systematic investigation of semantics of concurrent systems, it is worth considering all the possibilities for them to be equivalent. It results the better understanding of important properties of systems. On the other hand, for the practical purposes of specification and verification of the properties, it is of importance to have a number of suitable equivalence relations. In such a case, we are always able to choose the simplest possible viewpoint to the systems.

- Comparison equivalences provides one with understanding of their interrelations and nature, prevents of duplication of the known semantics. It leads to the decision which of the relations have to be additionally defined. By present time, a correlation on a number of important equivalences is not established.

- A problem of logical characterization of equivalences is of importance. It's decision offers one to consider behavior of systems in terms of formulas of temporal logics.

- Effective methods of systems' reduction which preserve their behavior modulo equivalences, are actual.

- During top-down design of systems and programs, one changes high structural abstraction level by low one. It is very important for the systems which had similar behaviors on one abstraction level to preserve this similarity on all the lower ones. Thus, one has to check which equivalence relations withstand an operation of refinement replacing some elementary components of systems by more complex structures. Presently, this question remains opened for a number of equivalences.

- Interrelations of concurrency and nondeterminism (conflict) like other peculiarities of concurrent systems often make an analysis of their behavior too intricate. Therefore, in the literature, a number of simpler subclasses of formal models have been proposed, which are suitable for practical purposes. It is actual to investigate equivalences on the mentioned subclasses to simplify checking of these relations, better understanding of their essential features and recovering complex behavioral information by simple one. This problem is also not well investigated.

- An important task is a development of equivalence notions for extensions of the known formal models (for example, by the notion of abstraction from invisible actions, adding a notion of time or a labelling of elementary events).

- A question about interconnections of equivalences defined in the frameworks of different formalisms is also actual. Its decision would provide one a possibility of changing specifications of a system in different models without changing of behavioral properties.

Purpose of the author's Ph.D. thesis [265] consists in the development of a number of methods and tools which can be useful for decision of the mentioned above problems.

Research methods are based on usage of the three important formal concurrency models: Petri nets, time Petri nets [192] and process algebras, their subclasses (sequential nets, strictly labelled nets, T-nets), extensions (nets with silent transitions, calculi of labelled processes) and apparatus of temporal logics.

New scientific achievements of the dissertation are the following.

1. On Petri nets both without and with silent transitions a wide set of behavioral equivalences have been proposed and investigated, which provide one with a possibility to abstract from structural and behavioral properties of modeled systems. These relations are distributed in the semantics from interleaving to true concurrency and from linear to branching time ones.

    - A diagram of interrelations of the mentioned equivalences has been obtained. A logical characterization of a number of equivalence notions has been proposed which can be used for treating behavior of concurrent systems in terms of temporal formulas.A method of effective net reduction has been described which preserves their behavior modulo equivalences.

    - Compositional aspects of behavior properties preservation for modeled systems have been explored.

    - Interrelation of equivalence notions on subclasses of nets has been established to simplify comparing their behavior and better understanding a nature of the relations.

2. On time Petri nets both without and with silent transitions a number of time, untime and regional equivalences have been investigated, which are able in different degree to take into account time aspects of behavior of modeled systems.

    - A correlation of the mentioned equivalences has been clarified. A regional characterization of time equivalences has been proposed, which simplifies check of the latter.

    - A compositional approach to the check of equivalences has been investigated.

    - Interrelation of equivalences on subclasses of time nets has been established.

3. Semantical equivalences of algebraic calculi and their extensions have been treated as well as their connections with net equivalence relations.

    - A new calculus of labelled nondeterministic concurrent processes $AFLP_2$ has been proposed, which is an extension of the known algebra AFP_2 (introduced by V.E. Kotov and L.A. Cherkasova) by labelling function. This labelling has offered a possibility of specifying much wider class of processes that of AFP_2.

- A complete and correct axiomatization of the equivalences w.r.t. denotational semantics of the mentioned algebras has been proposed as well as operational characterization of these equivalences which can be used for comparing these notions with behavioral relations.

- Interrelations of the algebraic and net equivalences has been established, which results a translation net specifications into algebraic and vice versa with preservation of behavior. It has integrated the advantages both nets and algebras.

- A term rewrite system for automatization of check of semantic equivalences has been defined. On it's basis, a computer program for check of formulas for equivalence has been written.

Practical usefulness of the Ph.D. thesis consists in the introducing of a number of equivalence relations for comparison and reduction of concurrent systems. Their systematic investigation helps to developer of a concurrent system to make a decision about most suitable model and an equality criteria (semantics).

Implementation of the results consists in their usage in the Laboratory of Theoretical Programming of IIS SD RAS during development a module for check of net equivalences for the system PEP (Programming Environment based on Petri nets) [52]. This system has appeared in the Institute of Informatics, Hildesheim, Germany. In addition, the author had written a computer program CANON which implements a checking of algebraic formulas by equivalence.

Publication of the results has been done in the following 13 articles on a theme of the dissertation: [252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 286].

During research work by theme of the Ph.D. thesis, the author participated in the following **scientific projects** (with the information about grants):

1. *An Investigation of Methods for Analysis and Verification of Concurrent Computational Systems, Programs and Processes.* Russian Fund for Basic Research (RFBR), grant 93-01-00986, headed by DR. V.A. NEPOMNIASCHY, 1993–1995. (Publications [253, 255, 256].)

2. *Models for Formal Semantics of Reactive Systems.* International Association for Promotion of Cooperation with Scientists from the Former Soviet Union (INTAS), grant 1010-CT93-0048, headed by DR. V.A. NEPOMNIASCHY, 1993–1995. (Publication [254].)

3. *Formal Methods in Design of Concurrent/Distributed Systems.* Volkswagen Stiftung (VS), grant I/70 564, headed by PROF. DR. EIKE BEST and DR. I.B. VIRBITSKAITE, 1995–1997. (Publications [257, 258, 259, 260, 261, 262, 263, 264, 286].)

4. *Development and Investigation of Semantical Methods and Tools of Specification and Verification of Concurrent Systems and Processes.* Russian Fund for Basic Research (RFBR), grant 96-01-01655, headed by DR. I.B. VIRBITSKAITE, 1995–1997. (Publications [259, 260, 261, 262, 286].)

5. *Methods and Tools for Verification and Analysis of Distributed Systems.* INTAS-RFBR, grant 95-0378, headed by DR. V.A. NEPOMNIASCHY, 1997–1999. (Publications [263, 264].)

6. International Soros Science Education Program (ISSEP), grant a97-683, 1997. (Publication [286].)

Approbation of the results has been done at the following conferences and schools:

1. 4th *International Conference on Applied Logics - 95 (AL'95)* and 2nd *Workshop on Non-Standard Logics and Logical Foundations of Computer Sciences (NSL'95),* Irkutsk, Russia, June 15–18, 1995.

2. 5th *Workshop on Concurrency, Specification and Programming - 96 (CSP'96),* Berlin, Germany, September 25–27, 1996.

3. 4th *Symposium on Logical Foundations of Computer Science - 97 (LFCS'97),* Yaroslavl, Russia, July 6–12, 1997.

4. *International Summer School Marktoberdorf on Computational Logic - 97 (CL'97),* Marktoberdorf, Germany, July 29 – August 10, 1997.

5. 4th *Workshop on Logic, Languages, Information and Computation - 97 (WoLLIC'97),* Fortaleza (Ceará), Brazil, August 19–22, 1997.

In addition, a number of lectures by theme of the Ph.D. thesis have been delivered at seminars of Institute of Informatics, Hildesheim, Germany, during the author's research work in the periods 01.12.95 – 29.02.96 and 01.03.97 – 30.04.97 within joint scientific project VS I/70 564. The results obtained have been also discussed at seminars of Laboratory of Theoretical Programming of IIS SD RAS.

Let us briefly describe the **contents** of the dissertation.

Chapter 1 is devoted to the investigation of equivalences on Petri nets.

In Section 1.1, the basic definitions are presented. Section 1.2 is about equivalence notions on nets without silent transitions, and Section 1.3 — on nets with silent transitions.

Chapter 2 is devoted to the investigation of equivalences on time Petri nets.

In Section 2.1, the basic definitions are presented. Section 2.2 is about equivalence relations on time nets without silent transitions, and Section 2.3 — on time nets with silent transitions.

Chapter 3 is devoted to the equivalences on process formulas.

In Section 3.1, the algebra AFP_2 is considered, and in Section 3.2 — the calculus $AFLP_2$.

In Appendix A (Proofs), a number of long proofs is moved.

In Appendix B (Description of the program CANON), a structure and principles of work of computer program CANON are described, which implements transformation of algebraic formulas into canonical form.

In Appendix C (Examples of work of the program CANON), several examples of transformation of AFP_2-formulas into canonical form using this program are presented.

The papers of the author are related with the themes of the chapters in the following way. By theme of Chapter 1 — [253, 254, 255, 257, 259, 261, 262, 263, 264], by Chapter 2 — [286] and by Chapter 3 — [252, 256, 258, 260].

In this survey, all the proofs, examples and appendices are omitted. The bibliography is not reduced, and therefore several items are possibly not cited here.

Chapter 1

Petri nets

Petri nets [228, 229, 230, 231, 243] is a suitable and powerful popular formal model both for specification and analysis of concurrent and distributed systems [222, 43, 44, 108]. Petri nets became widespread because of clear description such basic notions as causal dependence, non-determinism and concurrency. Their advantages are also good graphical representation and natural semantics of functioning based of rules of a "token game". Petri nets are actively used as a graphic tool for modeling of concurrent systems and processes at early stages of their development as a powerful analytical instrument in treating their behavioral properties.

In this chapter, equivalence notions are investigated on two classes of Petri nets: nets without silent transitions and nets with silent transitions (transitions can be labelled by a special symbol of invisible of silent action τ).

The main results of this chapter are the following.

- A number of known equivalence notions have been transferred into the framework of Petri nets from other formal models (for example, transition systems, event structures, behavioral structures, automata). As a result, a basis for their comparison and the decision which notions should be additionally defined has appeared.

- Several new equivalences for Petri nets have been introduced to obtain a complete enough set of them and filling empty places on the semantic coordinate plane in Figure 1. As a result, an ability for the choice of th simplest possible viewpoint to modeled systems has appeared. Moreover, the defining new semantics, which imitate in a new context that proposed earlier, can be avoided.

- Parametrized definitions of equivalences have been proposed, which offer their interpretation in a unified style and better understanding of nature of the relations.

- All the considered net equivalences have been compared, and a diagram of their interrelations is obtained which demonstrates differentiating power of the notions.

- A logical characterization of a number of equivalences has been given which offers investigation of behavior of nets using representation of their properties via formulas of temporal logics.

- The results by effective and semantic correct reduction of nets have been presented. The role of interrelations of equivalences in preservation of properties of a net to be reduced has been demonstrated.

- All the equivalences have been treated for preservation by refinement operation which corresponds to moving to lower structural abstraction level during construction of systems. As a result, it has been clarified, which equivalence relations can be used in top-down development of systems.

- A correlation of the equivalences on subclasses of nets has been investigated with the purposes of simplifying comparison of behavior of nets from the mentioned subclasses, better understanding a nature of equivalence relations and recovering complex behavioral information by the simpler one. It has been checked, which equivalences are differentiate well nets from the mentioned subclasses.

1.1 Basic definitions

In this section, in the main text of the dissertation, a number of basic definitions are introduced like that of multisets, labelled nets, marked nets, partially ordered sets (posets), labelled posets, partially ordered multisets (pomsets) [238], event structures, labelled event structures, multi event structures (i.e. event structures over multisets of actions) [211, 289, 292, 293], processes [34, 46, 150, 151] (based on causal nets) and branching processes [127, 169] (based on occurrence nets [152, 211, 229]).

1.2 Petri nets without silent transitions

Let us consider equivalences for Petri nets without silent transitions, which model systems all the actions of which are considered to be invisible for an external observer.

1.2.1 Basic equivalences

Let us see which mentioned in the literature equivalences fill the semantic coordinate plane in Figure 1.

- *Trace equivalences* (which respect only protocols of behavior of systems).

  - Interleaving trace equivalence ($\equiv_i$) has been introduced in [165, 166, 167] on languages. The definition is also in [216, 236, 241] on Petri nets, in [87, 130, 131, 136, 141, 142, 269, 272] — on event structures and in [126] — on common models of concurrent systems, and in [244].
  - Step trace equivalence ($\equiv_s$). A definition for Petri nets can be found in [236, 241], on event structures — in [130, 131, 136, 141, 142, 269, 272].
  - Partial word trace equivalence ($\equiv_{pw}$). The notion has been defined [153] on pomsets and in [280] — on event structures.
  - Pomset trace equivalence ($\equiv_{pom}$). The corresponding semantics has been introduced in [238, 239, 240]. The equivalence has been defined in [154, 241]on Petri nets and in [130, 131, 136, 141, 142, 269, 272] — on event structures.

- *Usual bisimulation equivalences* (which respect branching structure of behavior of systems).

– Interleaving bisimulation equivalence ($\underleftrightarrow{}_i$) has been introduced in [223] on automata on the basis of observational equivalence [194, 176]. The definition can be also found in [224] on automata, in [1, 2, 16, 17, 26, 38, 39, 115, 154, 216, 241] — on Petri nets, in [87, 130, 131, 136, 141, 142, 269, 272, 277, 278, 282, 284] — on event structures, and in [126] — on common models of concurrent systems.

– Step bisimulation equivalence ($\underleftrightarrow{}_s$) has been introduced in [214]. The definition on Petri nets is in [1, 2, 16, 17, 28, 154, 236, 241], and on event structures — in [130, 131, 136, 141, 142, 277, 278, 282, 284].

– Partial word bisimulation equivalence ($\underleftrightarrow{}_{pw}$) has been introduced in [277, 278] on event structures. The definition on Petri nets can be found in [16, 17], and that for event structures — also in [282, 284].

– Pomset bisimulation equivalence ($\underleftrightarrow{}_{pom}$) has been defined in [27, 154]. The definition on Petri nets is in [1, 2, 16, 17, 26, 38, 39, 241], and on event structures — in [130, 131, 136, 141, 142, 277, 278, 282, 284].

– Process bisimulation equivalence ($\underleftrightarrow{}_{pr}$) has been proposed in [16] on Petri nets. The definition is also in [17].

- *ST-bisimulation equivalences* (which respect the duration of transition occurrences in behavior of systems).

– Interleaving ST-bisimulation equivalence ($\underleftrightarrow{}_{iST}$) has been introduced in [154] on Petri nets. The definition for Petri nets can be also found in [26, 115], and for event structures — in [141, 142, 277, 278, 282, 284].

– Partial word ST-bisimulation equivalence ($\underleftrightarrow{}_{pwST}$) has been proposed in [277, 278] on event structures. The definition can be also found in [282, 284].

– Pomset ST-bisimulation equivalence ($\underleftrightarrow{}_{pomST}$) has been introduced in [277, 278] on event structures. The definition is also in [282, 284].

- *History preserving bisimulation equivalences* (which respect the "past" or "history" of behavior of systems).

– Pomset history preserving bisimulation equivalence ($\underleftrightarrow{}_{pomh}$) has been proposed in [247] on behavioral structures under the name "behavioral structure bisimulation equivalence".

In [130], the equivalence has been introduced on event structures and called "history preserving bisimulation equivalence".

In [109], in the framework of event structures "causal trees observable equivalence" has been defined, the coincidence of which with history preserving bisimulation equivalence has been proved in [270, 110, 111].

In [120], NMS (Nondeterministic Measurement System) equivalence has been defined which is also coincide with history preserving bisimulation one.

In [4, 233, 83], it has been demonstrated that history preserving bisimulation equivalence coincide with "mixed orders equivalence" on event structures from [122].

In [95], "back-forth pomset bisimulation equivalence" on event structures has been proposed. This is also coincide with history preserving bisimulation one.

In [38], the definition has been introduced on Petri nets. In the paper, the notion has been called "fully concurrent bisimulation equivalence".

The definition for Petri nets can be also found in [26, 39, 113, 114, 115, 132, 241], for event structures — in [131, 133, 134, 135, 136, 141, 142, 277, 278, 279, 282, 284], and for behavioral structures — in [266].

- *Conflict preserving equivalences* (which fully respect conflicts in systems).

  - Occurrence conflict preserving equivalence ($\equiv_{occ}$) has been proposed in [211, 154] on Petri nets.

- *Isomorphism* (i.e. coincidence of systems up to renaming of their components, $\simeq$) has been introduced in [228].

In the papers [73, 143, 144, 145, 160, 267, 274, 275, 276, 280, 282, 283, 285], a set of equivalences has been considered which are in between trace and bisimulation relations, i.e. within from "linear time" to "branching time" semantics [146].

A notion of distributed bisimulation has been introduced in [94, 162].

The equivalences based on "locations" has been studied in [32, 33, 82, 181, 205, 107]. A review of the set of relations based on "locations" and "causes" is in [83, 182]. Similar equivalences has been investigated in [83].

Reviews of equivalences has been done also in [81, 124, 235]. Pomset relations has been considered in [245].

In this section, we transfer all the basic equivalence notions into the framework of Petri nets. Moreover, we introduce a number of new basic equivalences which complete the set of the already known ones in the mentioned above semantics. These are process trace ($\equiv_{pr}$), process ST-bisimulation ($\underline{\leftrightarrow}_{prST}$), process history preserving bisimulation ($\underline{\leftrightarrow}_{prh}$), multi event structure (MES) conflict preserving ($\equiv_{mes}$) equivalences. The symbols of the new relations are in rectangles in Figure 1.1.

We compare all the equivalences. As a result, we obtain a diagram of their interrelations as a oriented graph with edges corresponding to the relation "stronger than".

Comparing basic equivalences

The following theorem which establishes interrelations all the basic equivalences.

In the following, the symbol '$\_$' will denote "nothing", and the signs of equivalences subscribed by it are considered as that of without any subscription.

Theorem 1.2.1 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\rightarrow \in \{\equiv, \underline{\leftrightarrow}, \simeq\}$ *and* $\star, \star\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST, prST, pomh, prh, mes, occ\}$. *For nets* N *and* N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\!\!\!\rightarrow_{\star\star} N'$ *iff there exists a directed path from* $\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\rightarrow_{\star\star}$ *in the graph in Figure 1.2.*

1.2.2 Back-forth equivalences

Back-forth bisimulation equivalences are based on the idea that bisimulation relation do not only require systems to simulate each other behavior in the forward direction (as usually) but also when going back in history. They are closely connected with equivalences of logics with past modalities [88, 139, 187, 189] and "true concurrency" logics [225, 226, 227].

These equivalence notions have been initially introduced in [210] in the framework of transition systems. It has been shown that back-forth variant ($\underline{\leftrightarrow}_{ibif}$) of interleaving bisimulation equivalence coincide with ordinary $\underline{\leftrightarrow}_i$.
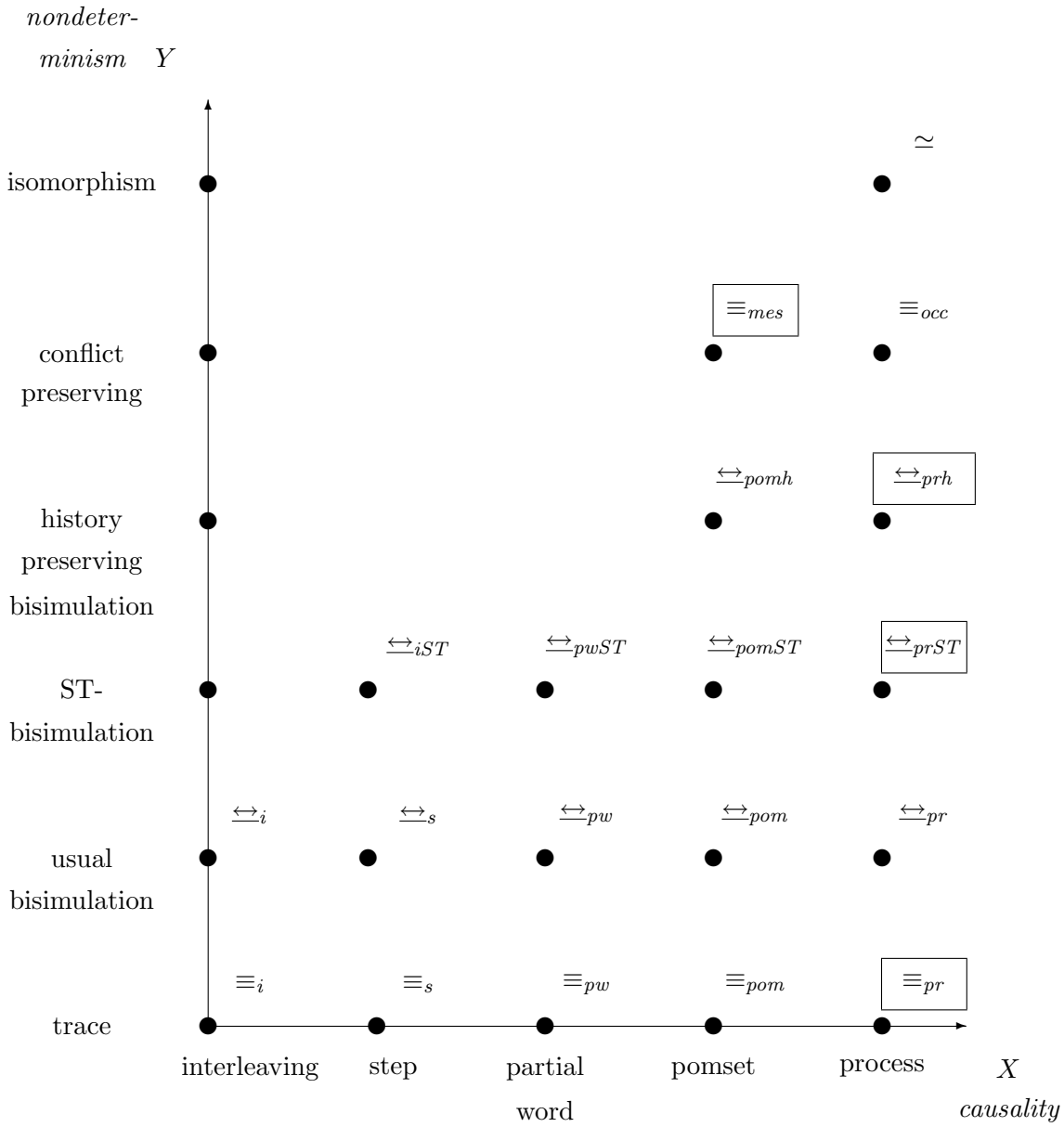
Figure 1.1: Classification of basic equivalences

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$$\simeq$$
$$\downarrow$$
$$\equiv_{mes} \longleftarrow \equiv_{occ}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\underline{\leftrightarrow}_{pomh} \longleftarrow \underline{\leftrightarrow}_{prh}$$
$$\downarrow \qquad\qquad \downarrow$$
$$\underline{\leftrightarrow}_{iST} \longleftarrow \underline{\leftrightarrow}_{pwST} \longleftarrow \underline{\leftrightarrow}_{pomST} \longleftarrow \underline{\leftrightarrow}_{prST}$$
$$\downarrow \qquad\quad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$
$$\underline{\leftrightarrow}_i \longleftarrow \underline{\leftrightarrow}_s \longleftarrow \underline{\leftrightarrow}_{pw} \longleftarrow \underline{\leftrightarrow}_{pom} \longleftarrow \underline{\leftrightarrow}_{pr}$$
$$\downarrow \qquad\quad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$
$$\equiv_i \longleftarrow \equiv_s \longleftarrow \equiv_{pw} \longleftarrow \equiv_{pom} \longleftarrow \equiv_{pr}$$
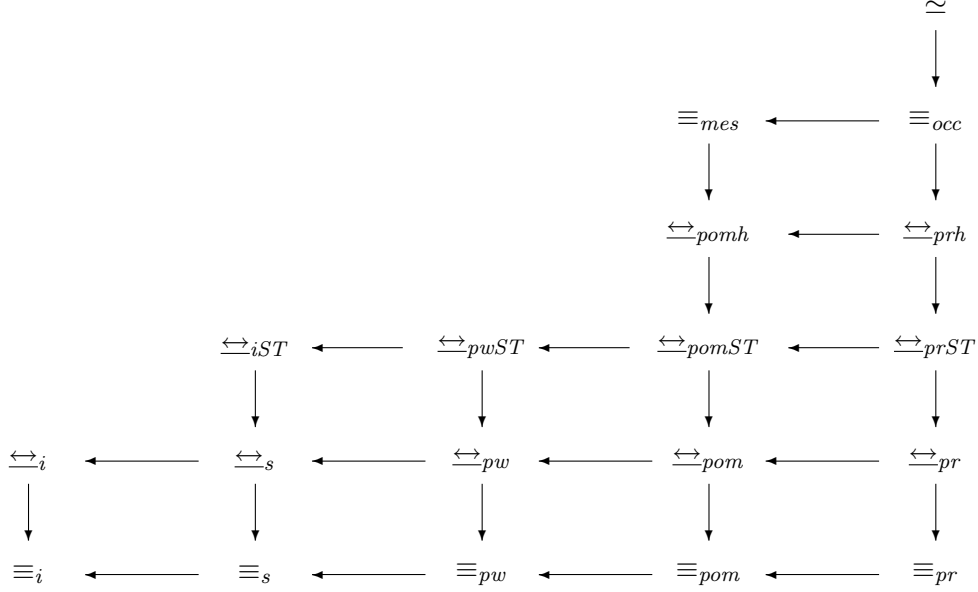
Figure 1.2: Interrelations of the equivalences

In [105, 95, 96, 97], the new variants of step ($\underline{\leftrightarrow}_{sbsf}$), partial word ($\underline{\leftrightarrow}_{pwbpwf}$) and pomset ($\underline{\leftrightarrow}_{pombpomf}$) back-forth bisimulation equivalences have been defined in the framework of prime event structures. The equivalence notions have been compared with usual, ST- and history preserving bisimulation equivalences. It was shown that $\underline{\leftrightarrow}_{pomST}$ implies $\underline{\leftrightarrow}_{sbsf}$. The coincidence of $\underline{\leftrightarrow}_{pombpomf}$ and $\underline{\leftrightarrow}_{pomh}$ has been proved, giving rise to a new, logical characterization of the latter.

In [233], the new idea of differentiating the kinds of back and forth simulations has appeared (following this idea, it is possible, for example, to define step back pomset forth bisimulation equivalence ($\underline{\leftrightarrow}_{sbpomf}$)). The set of all possible back-forth equivalence notions has been proposed in interleaving, step, partial word and pomset semantics. Two new notions which do not coincide with known ones has been obtained: step back partial word forth ($\underline{\leftrightarrow}_{sbpwf}$) and step back pomset forth ($\underline{\leftrightarrow}_{sbpomf}$) bisimulation equivalences.

Back-forth bisimulation equivalences have been also considered in the papers [40, 170, 171, 207, 215].

In this section, we extend the set of back-forth equivalences from [233] by new relations for process semantics and obtain as a result two new notions which cannot be reduced to the known relations: step back process forth ($\underline{\leftrightarrow}_{sbprf}$)and pomset back process forth ($\underline{\leftrightarrow}_{pombprf}$) bisimulation equivalences.

We add the results of [97, 233] and establish interrelations between back-forth and basic equivalences. In particular, we prove that $\underline{\leftrightarrow}_{pomST}$ implies $\underline{\leftrightarrow}_{sbpomf}$, and $\underline{\leftrightarrow}_{prST}$ implies $\underline{\leftrightarrow}_{sbprf}$. We also demonstrate a coincidence of $\underline{\leftrightarrow}_{prbprf}$ and $\underline{\leftrightarrow}_{prh}$.

Comparing back-forth bisimulation equivalences

Let us consider interrelations of back-forth bisimulation equivalences.

Proposition 1.2.1 *Let $\star \in \{i, s, pw, pom, pr\}$. For nets N and N' the following holds:*

1. $N \underline{\leftrightarrow}_{pwb\star f} N' \Leftrightarrow N \underline{\leftrightarrow}_{pomb\star f} N'$;
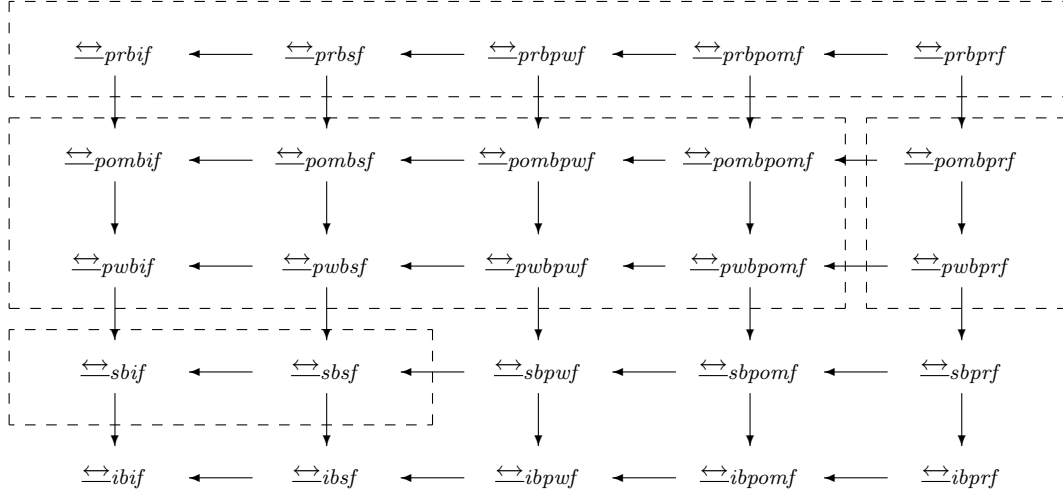
Figure 1.3: Merging of back-forth bisimulation equivalences
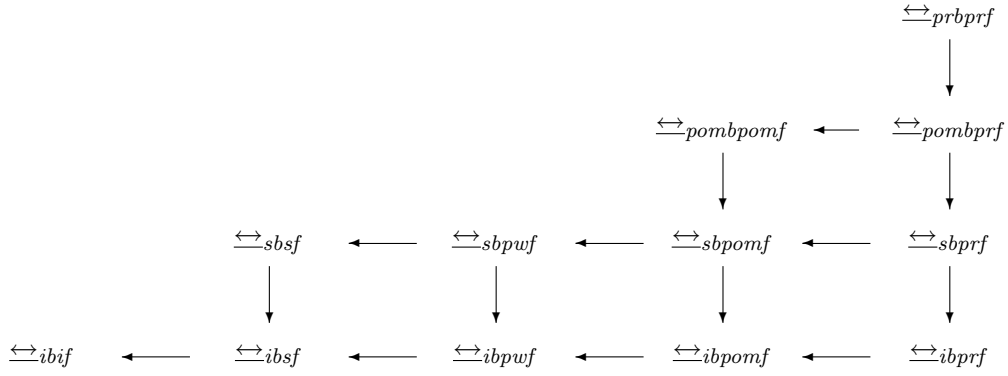
Figure 1.4: Interrelations of back-forth bisimulation equivalences

2. $N\underline{\leftrightarrow}_{\star bif}N' \ \Leftrightarrow \ N\underline{\leftrightarrow}_{\star b\star f}N'$.

In Figure 1.3, dashed lines embrace coinciding back-forth bisimulation equivalences.

Hence, interrelations of back-forth bisimulation equivalences may be represented by graph in Figure 1.4.

Comparing back-forth bisimulation equivalences with basic equivalences

Let us consider interrelations of back-forth bisimulation equivalences with basic equivalences.

Proposition 1.2.2 *Let $\star \in \{i, s, pw, pom, pr\}$ and $\star\star \in \{pom, pr\}$. For nets N and N' the following holds:*

1. $N\underline{\leftrightarrow}_{ib\star f}N' \ \Leftrightarrow \ N\underline{\leftrightarrow}_{\star}N'$;

2. $N\underline{\leftrightarrow}_{\star\star b\star\star f}N' \ \Leftrightarrow \ N\underline{\leftrightarrow}_{\star\star h}N'$;

3. $N\underline{\leftrightarrow}_{\star\star ST}N' \ \Rightarrow \ N\underline{\leftrightarrow}_{sb\star\star f}N'$.

16

$$\underline{\simeq}$$

$$\equiv_{mes} \longleftarrow \equiv_{occ}$$

$$\underline{\leftrightarrow}_{pomh} \qquad \underline{\leftrightarrow}_{prh}$$

$$\underline{\leftrightarrow}_{pombprf}$$

$$\underline{\leftrightarrow}_{iST} \longleftarrow \underline{\leftrightarrow}_{pwST} \longleftarrow \underline{\leftrightarrow}_{pomST} \longleftarrow \underline{\leftrightarrow}_{prST}$$

$$\underline{\leftrightarrow}_{sbsf} \longleftarrow \underline{\leftrightarrow}_{sbpwf} \longleftarrow \underline{\leftrightarrow}_{sbpomf} \longleftarrow \underline{\leftrightarrow}_{sbprf}$$

$$\underline{\leftrightarrow}_{i} \longleftarrow \underline{\leftrightarrow}_{s} \longleftarrow \underline{\leftrightarrow}_{pw} \longleftarrow \underline{\leftrightarrow}_{pom} \longleftarrow \underline{\leftrightarrow}_{pr}$$

$$\equiv_{i} \longleftarrow \equiv_{s} \longleftarrow \equiv_{pw} \longleftarrow \equiv_{pom} \longleftarrow \equiv_{pr}$$
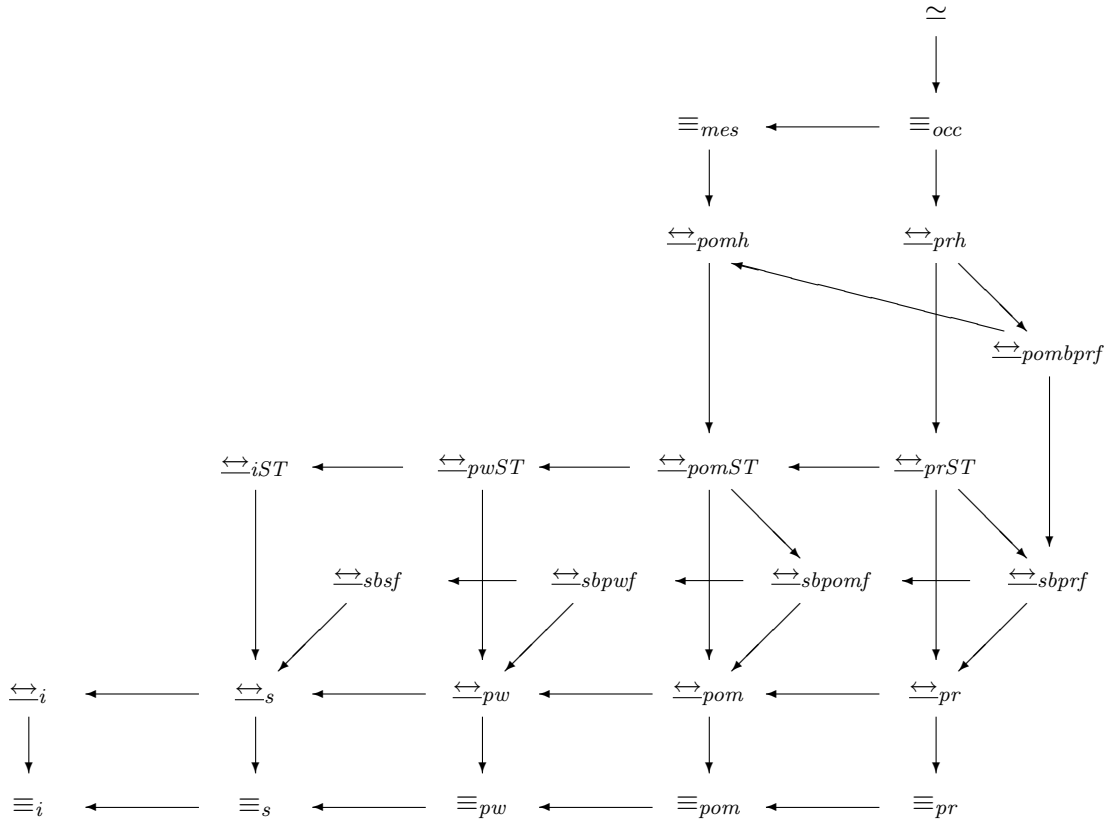
Figure 1.5: Interrelations of back-forth bisimulation equivalences with basic equivalences

Theorem 1.2.2 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\twoheadrightarrow \in \{\equiv, \underleftrightarrow{\;}, \simeq\}$ *and* $\star, \star\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST,$ $prST, pomh, prh, mes, occ, sbsf, sbpwf, sbpomf, sbprf, pombprf\}$. *For nets* N *and* N' $N \leftrightarrow_\star N'$ $\Rightarrow N \leftrightarrow\!\!\!\twoheadrightarrow_{\star\star} N'$ *iff in the graph in Figure 1.5 there exists a directed path from* $\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\twoheadrightarrow_{\star\star}$.

Logical characterization

In this section, we demonstrate that several important back-forth bisimulation equivalences coincide with that of temporal logics having past modalities. These results provide a logical characterization of bisimulation equivalences (or, dually, an operational characterization of logical ones).

Logic HML Logic of Hennessy-Milner (HML) has been introduced in [164] on transition systems for logical description of interleaving bisimulation equivalence.

Standard logical equivalence of the logic on Petri nets is denoted by $=_{HML}$. We call a Petri net to be *image finite* one if its' processes can be extended by any particular action only in finite number of ways.

Theorem 1.2.3 *[164] For image-finite nets* N *and* N' $N \underleftrightarrow{}_i N' \Leftrightarrow N \underleftrightarrow{}_{ibif} N' \Leftrightarrow N =_{HML} N'$.

Logic $PBFL$ Pomset back-forth logic ($PBFL$) has been proposed in [88] on event structures for logical description of pomset back pomset forth bisimulation equivalence.

Standard logical equivalence of the logic on Petri nets is denoted by $=_{PBFL}$.

Theorem 1.2.4 *[88] For image-finite nets* N *and* N' $N \underleftrightarrow{}_{pomh} N' \Leftrightarrow N \underleftrightarrow{}_{pombpomf} N' \Leftrightarrow$ $N =_{PBFL} N'$.

Logic $PrBFL$ We introduce the new process back-fort logic ($PrBFL$) for logical description of process back process forth bisimulation equivalence.

Standard logical equivalence of the logic on Petri nets is denoted by $=_{PrBFL}$.

Theorem 1.2.5 *For image-finite nets* N *and* N' $N \underleftrightarrow{}_{prh} N' \Leftrightarrow N \underleftrightarrow{}_{prbprf} N' \Leftrightarrow N =_{PrBFL} N'$.

1.2.3 Place equivalences

Place bisimulation equivalences have been initially introduced in [1] on the basis of definition from [219, 220, 221]. Place bisimulations are relations over places instead of markings or processes. The relation on markings is obtained using the *"lifting"* of relation on places.

The main application of place bisimulation equivalences is an effective behavior preserving reduction technique for Petri nets based on them.

In [1, 2], interleaving place bisimulation equivalence ($\sim_i$)has been proposed. In these papers, also strict interleaving place bisimulation equivalence ($\approx_i$) has been defined, by imposing the additional requirement stating that corresponding transitions of nets must be related by bisimulation. The question about possibility to introduce history preserving place bisimulation equivalence was addressed.

In [16, 17], step ($\sim_s$), partial word ($\sim_{pw}$), pomset ($\sim_{pom}$), process ($\sim_{pr}$) place bisimulation equivalences and their strict analogues ($\approx_s$, $\approx_{pw}$, $\approx_{pom}$, $\approx_{pr}$) have been proposed. The coincidence of $\sim_i$, $\sim_s$ and $\sim_{pw}$ has been established. Also it was shown that all strict bisimulation equivalences coincide with $\sim_{pr}$. Therefore, we have only three different equivalences: $\sim_i$, $\sim_{pom}$ and $\sim_{pr}$. In addition, in these papers the polynomial algorithm of net reduction was

$$\approx_i \longleftarrow \approx_s \longleftarrow \approx_{pw} \longleftarrow \approx_{pom} \longleftarrow \approx_{pr}$$

$$\sim_i \longleftarrow \sim_s \longleftarrow \sim_{pw} \longleftarrow \sim_{pom} \longleftarrow \sim_{pr}$$
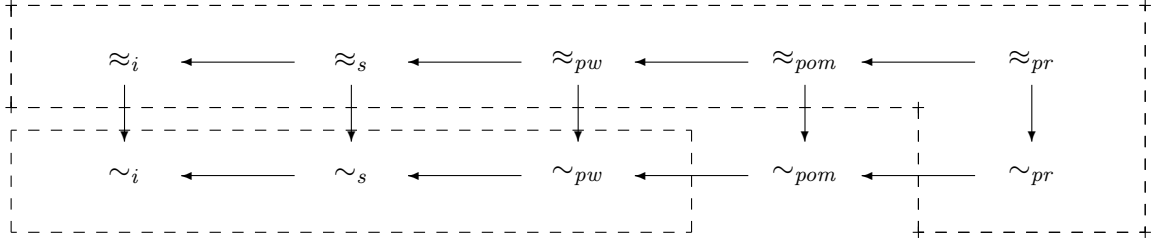
Figure 1.6: Merging of place bisimulation equivalences

$$\sim_i \longleftarrow \sim_{pom} \longleftarrow \sim_{pr}$$

Figure 1.7: Interrelations of place bisimulation equivalences

proposed which preserves the behavior of a net (i.e. the initial and reduced nets are bisimulation equivalent).

In this section, we compare place bisimulation equivalences with back-forth bisimulation and basic ones and extend the results of [16, 17].

In particular, we prove that $\sim_{pr}$ implies $\underline{\leftrightarrow}_{prh}$ and answer the question from [1]: $\sim_{pr}$ is strict enough to preserve the "histories" of a net functioning. Hence, it is no sense to define history preserving place bisimulation equivalence.

Since ST- and history preserving bisimulation equivalences are consequences of $\sim_{pr}$, the algorithm of net reduction from [16, 17], based on this equivalence, preserves the *timed traces* [154] of the initial net (since ST-bisimulation equivalences are *real time consistent* [154]) and "histories" of its functioning (since history preserving bisimulation equivalences respect the "past" of processes).

Comparing place bisimulation equivalences

Let us consider interrelations of place bisimulation equivalences.

Proposition 1.2.3 *[16, 17] For nets N and N':*

1. $N \sim_i N' \iff N \sim_{pw} N'$;

2. $N \sim_{pr} N' \iff N \approx_i N' \iff N \approx_{pr} N'$.

In Figure 1.6, dashed lines embrace coinciding place bisimulation equivalences.

Hence, interrelations of place bisimulation equivalences may be represented by graph in Figure 1.7.

Comparing place bisimulation equivalences with basic equivalences and back-forth bisimulation ones

Let us consider interrelations of place bisimulation equivalences with basic equivalences and back-forth bisimulation equivalences.

Proposition 1.2.4 *For nets N and N' $N \sim_{pr} N' \implies N \underline{\leftrightarrow}_{prh} N'$.*
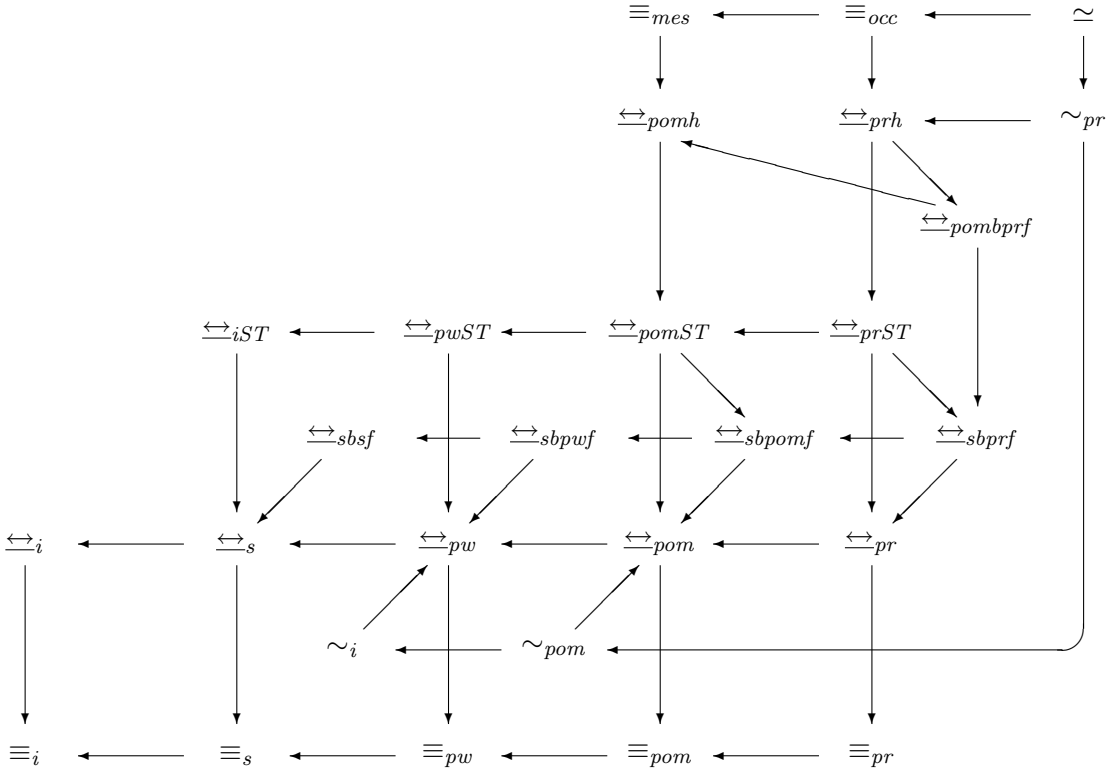
Figure 1.8: Interrelations of place bisimulation equivalences with basic equivalences and back-forth bisimulation equivalences

Theorem 1.2.6 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \sim, \simeq\}$, $\star, \star\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST$
$prST, pomh, prh, mes, occ, sbsf, sbpwf, sbpomf, sbprf, pombprf\}$. *For nets* N *and* N' $N \leftrightarrow_\star N'$
$\Rightarrow N \leftrightarrow\!\!\!\leftrightarrow_{\star\star} N'$ *iff in the graph in Figure 1.8 there exists a directed path from* $\leftrightarrow\!\!\!\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\leftrightarrow_{\star\star}$.

Reduction of nets based on place bisimulation equivalences

In the literature, a lot o reduction methods for Petri nets have been proposed (see, for example, [209]). Unfortunately, many of them are local, not always effective, or they preserve few behavioral properties [16, 17, 15].

Place bisimulations offer an effective, global and semantic correct simplification of nets [16, 17]. The basic idea consists in considering so called place "autobisimulation" equivalences, i.e. the equivalences between a net and itself. After this, one is to merge equivalent places and reduce superfluous transitions and arcs.

In the papers [16, 17], the algorithm is based on $\sim_i$.

The following results concerning $\sim_{pom}$ and $\sim_{pr}$ are known.

We cannot use $\sim_{pom}$ for simplification of nets, since in [16, 17], an example is presented when for a net N the following holds: $N \not\sim_{pom} N/_{\sim_{pom}}$.

Since $\sim_{pr}$ coincide with $\approx_i$, we can slightly modify an algorithm for $\mathcal{R}_i$ to obtain $\mathcal{R}_{pr}$: we shall check by bisimulation all the pairs of transitions which appear during treating for the transfer property. The complexity of the algorithm will be the same. Thus, it is possible to reduce net effectively modulo $\sim_{pr}$.

We have established interrelations of $\sim_{pr}$ with the other equivalences, and this provides us with the following important results.

- Since $\sim_{pr}$ implies $\underline{\leftrightarrow}_{prh}$ and $\underline{\leftrightarrow}_{prST}$, a reduced net has the same histories of behavior and *timed traces* [154] as the initial one.

- Since $\underline{\leftrightarrow}_{prh}$ coincide with $=_{PrBFL}$ then all the properties described by formulas of logic $PrBFL$ are preserved in the reduced net.

1.2.4 Preservation of equivalences by refinements

When considering the nets related by some equivalence, the question appears, whether this equivalence is preserved by some equal transformation of the nets. During top-down development of distributed systems, a transformation called *refinement* is applied [106]. In accordance to the concept of refinement, the actions (or states) which were considered to be atomic on one abstraction level, become having some internal structure on the lower level.

By now, a notion of refinement is a subject of deep research as in the framework of net models [57, 110, 112, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 157, 213, 237, 273, 274, 275, 276, 280, 284], as in the process algebras [6, 7, 8, 208].

In this section, we consider transition refinement, in accordance to which the transitions of a net which are labeled by some particular action are replaced by subnets of a special kind: so-named, SM-nets, a subclass of state machine nets or S-nets [39].

Let us note that in the literature, preservation by SM-refinements has been proved only for $\underline{\leftrightarrow}_{iST}$ [114, 115] and $\underline{\leftrightarrow}_{pomh}$ [39]. For $\simeq$, the property is obvious. In [106], preservation of $\equiv_{pom}$ has been demonstrated, but it has been done on the other model and for another refinement operator. Analogously, the property for $\underline{\leftrightarrow}_{pwST}$ and $\underline{\leftrightarrow}_{pomST}$ has been established in [278, 282], but it has been done in the framework of event structures, and another type of refinement has been applied (which, for example, does not permit conflicts in a substituted event structure).
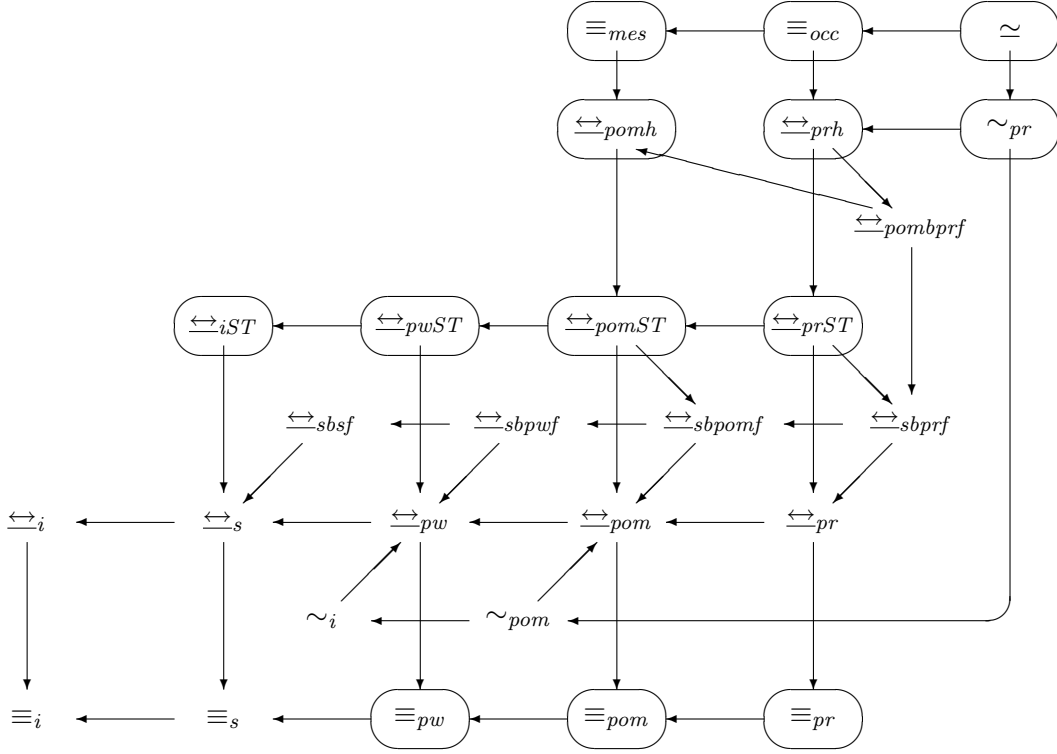
Figure 1.9: Preservation of the equivalences by SM-refinements

The new results, considered in this section, are proofs of preservation by SM-refinements of $\equiv_\star, \underline{\leftrightarrow}_{\star ST}$, $\star \in \{pw, pom, pr\}$ and $\equiv_{mes}$, $\equiv_{occ}$. Very important result is preservation of $\sim_{pr}$ by SM-refinements. The consequence is the possibility to use the reduction method for nets based on this equivalence during top-down design of systems. In addition, using counterexamples, we demonstrate that none from the other considered equivalences have the property.

Theorem 1.2.7 *Let* $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \sim, \simeq\}$ *and* $\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST, prST, pomh, prh, mes, occ, sbsf, sbpwf, sbpomf, sbprf, pombprf\}$. *For nets* N, N' *s.t.* $a \in l_N(T_N) \cap l_{N'}(T_{N'}) \cap Act$ *and SM-net* D *the following holds:* $N \leftrightarrow_\star N' \Rightarrow ref(N, a, D) \leftrightarrow_\star ref(N', a, D)$ *iff the equivalence* $\leftrightarrow_\star$ *is in oval in Figure 1.9.*

1.2.5 The equivalences on subclasses of Petri nets

In the literature, a number of Petri net subclasses have been proposed via imposing some restrictions on the initial definition of nets. Thus, the considered equivalences can become more dependent each of another, or the can even merge on such nets.

The results for net subclasses can be found in [1, 89, 241, 294], and on restrictions of other formal models — in [12, 126, 269, 271, 272].

These results can be used as for simplification of checking nets by an equivalence as for recovering some complex information about their behavior by simpler one. For example, in [180], it was described how to recover partial languages of P/T nets by the set of their step traces.

Interrelations of concurrency and nondeterminism essentially complicate analysis of systems. Therefore, in Petri net theory, the subclasses of nets have been introduced in which there is the only aspect from the two mentioned above.

In this section, we shall consider such subclasses of Petri nets like sequential nets [39], where parallel firing of transitions is impossible, and T-nets [246], where conflict transitions do not exist.

It is well-known that labelled Petri net have more modeling power than that without labeling. Therefore, it is interesting to consider just another subclass of Petri nets: strictly labeled nets [39], where no two different transitions may be labeled by the same action. Consequently, these nets may be considered as unlabeled, if identify the names of transitions with their labels.

Let us note that, in the literature, only $\underleftrightarrow{}_i, \underleftrightarrow{}_{pom}$ and $\underleftrightarrow{}_{pomh}$ have been treated on sequential nets [39]. No equivalences have been compared on T-nets. In [241], on strictly labeled nets, only a little set of equivalences have been compared which has contained $\equiv_\star, \underleftrightarrow{}_\star, \star \in \{i, s, pom\}$ and $\underleftrightarrow{}_{pomh}$.

Thus, interrelations of equivalences from such a wide set are investigated for the first time, and it can be considered as a new result.

The equivalences on sequential nets

Let us consider the equivalences on sequential nets, where no two transitions can be fired concurrently.

Proposition 1.2.5 *For sequential nets N and N':*

1. $N \equiv_i N' \Leftrightarrow N \equiv_{pom} N'$;

2. *[241]* $N \underleftrightarrow{}_i N' \Leftrightarrow N \underleftrightarrow{}_{pomh} N'$;

3. $N \underleftrightarrow{}_{pr} N' \Leftrightarrow N \underleftrightarrow{}_{pombprf} N'$;

4. $N \underleftrightarrow{}_{pr} N' \Leftrightarrow N \underleftrightarrow{}_{pombprf} N'$;

5. $N \sim_i N' \Leftrightarrow N \sim_{pom} N'$.

In Figure 1.10, dashed lines embrace the equivalences coinciding on sequential nets.

Theorem 1.2.8 *Let $\leftrightarrow, \leftrightarrow\kern-0.6em\leftrightarrow \in \{\equiv, \underleftrightarrow{}, \sim, \simeq\}$, $\star, \star\star \in \{_-, i, pr, prST, prh, mes, occ\}$. For sequential nets N and N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\kern-0.6em\leftrightarrow_{\star\star} N'$ iff in the graph in Figure 1.11 there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow\kern-0.6em\leftrightarrow_{\star\star}$.*

The equivalences on strictly labeled nets

Let us consider the equivalences on strictly labeled nets where no two different transitions may have the same label.

Proposition 1.2.6 *Let $\star \in \{i, s, pw, pom, pr\}$. For strictly labeled nets N and N':*

1. $N \equiv_\star N' \Leftrightarrow N \underleftrightarrow{}_\star N'$;

2. $N \underleftrightarrow{}_s N' \Leftrightarrow N \underleftrightarrow{}_{iST} N'$.

In Figure 1.12, dashed lines embrace the equivalences coinciding on strictly labeled nets.
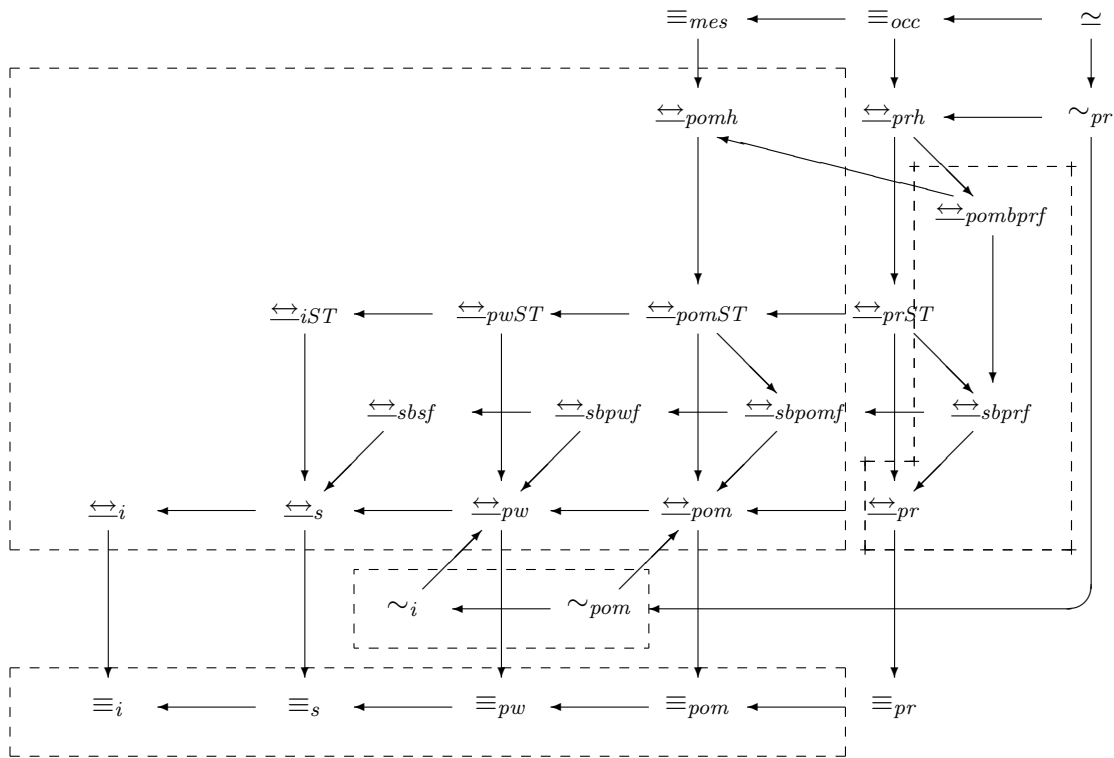
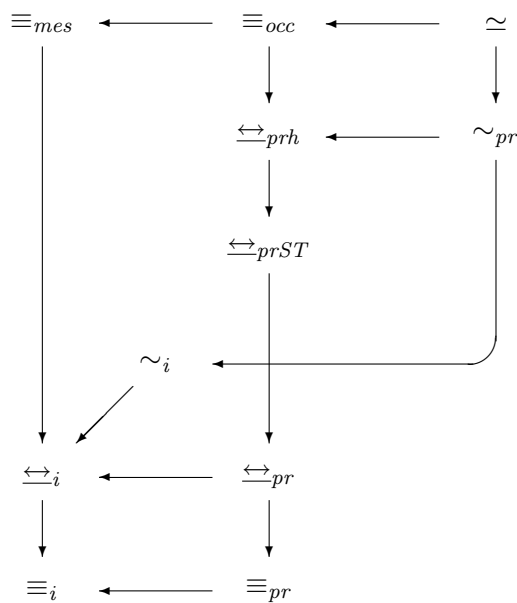Figure 1.10: Merging of the equivalences on sequential nets

Figure 1.11: Interrelations of the equivalences on sequential nets

Figure 1.12: Merging of the equivalences on strictly labeled nets

The equivalences on T-nets

Let us consider the equivalences on T-nets where no two transitions may have common input or output place (i.e. to be in the forward or the backward conflict). Merging of equivalences is obtained on even more restricted subclass: T-nets without autoconcurrency, where, in addition to the previous requirement, no two equally labeled transitions cam be fired in parallel at any reachable marking.

Proposition 1.2.7 *For T-nets without autoconcurrency N and N' $N \equiv_i N' \Leftrightarrow N \underline{\leftrightarrow}_{iST} N'$.*

In Figure 1.13, dashed lines embrace the equivalences coinciding on T-nets without autoconcurrency.

1.3   Petri nets with silent transitions

In this section, we consider an extension of Petri nets by silent transitions. Silent transitions are that of labelled by special *silent* action τ which represents an internal activity of a system to be modeled and it is invisible for an external observer which examines the behavior of the system. Nevertheless, silent action can influence the behavior. It is well-known that Petri nets with silent transitions are more powerful than usual ones.

Equivalences which abstract of silent actions are called τ-*equivalences* (these are labelled by the symbol τ to distinguish them of relations not abstracting of silent actions).

25

$$\equiv_{mes} \longleftarrow \equiv_{occ} \longleftarrow \simeq$$

$$\underleftrightarrow{}_{pomh} \qquad \underleftrightarrow{}_{prh} \longleftarrow \sim_{pr}$$

$$\underleftrightarrow{}_{pombprf}$$

$$\underleftrightarrow{}_{iST} \longleftarrow \underleftrightarrow{}_{pwST} \longleftarrow \underleftrightarrow{}_{pomST} \longleftarrow \underleftrightarrow{}_{prST}$$

$$\underleftrightarrow{}_{sbsf} \longleftarrow \underleftrightarrow{}_{sbpwf} \longleftarrow \underleftrightarrow{}_{sbpomf} \longleftarrow \underleftrightarrow{}_{sbprf}$$

$$\underleftrightarrow{}_{i} \longleftarrow \underleftrightarrow{}_{s} \qquad \underleftrightarrow{}_{pw} \longleftarrow \underleftrightarrow{}_{pom} \longleftarrow \underleftrightarrow{}_{pr}$$

$$\sim_{i} \longleftarrow \sim_{pom}$$

$$\equiv_{i} \longleftarrow \equiv_{s} \longleftarrow \equiv_{pw} \longleftarrow \equiv_{pom} \longleftarrow \equiv_{pr}$$
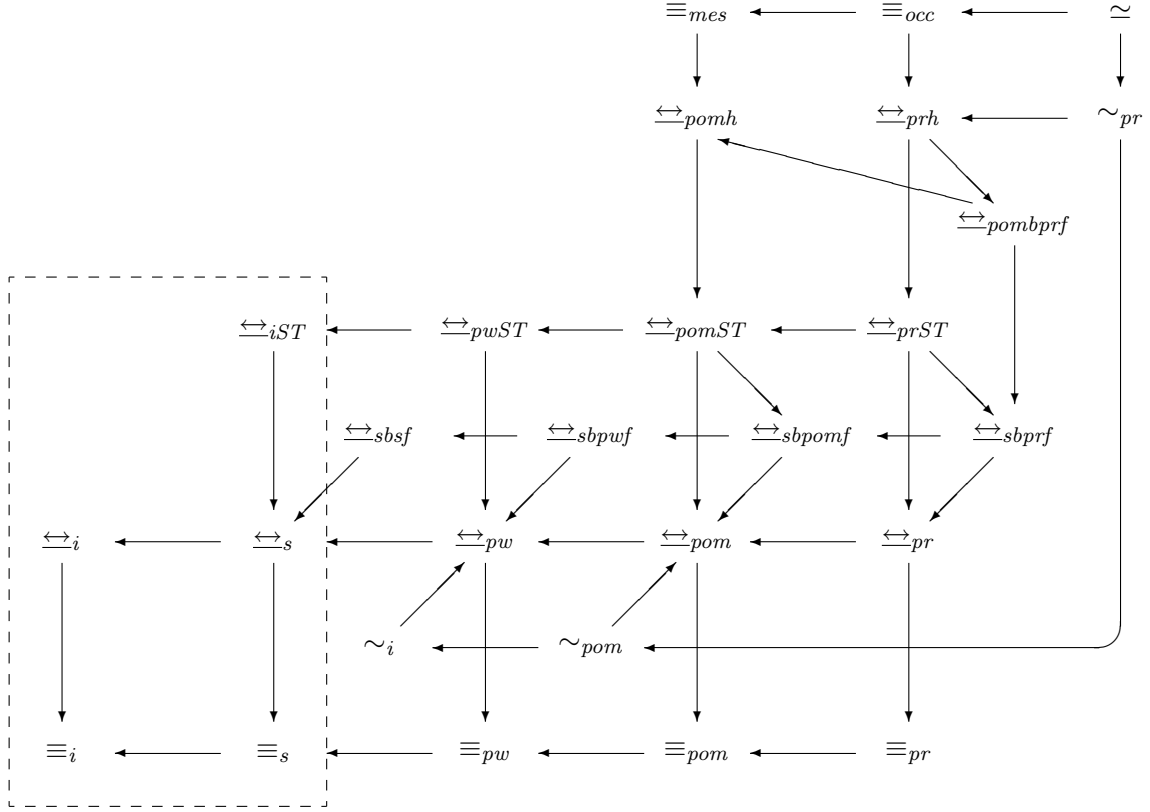
Figure 1.13: Merging of the equivalences on T-nets without autoconcurrency

τ-equivalences coincide with the considered before ones on Petri nets without silent transitions. Since usual Petri nets are a subclass of that with silent transitions, several implications between usual equivalences can become not valid anymore between their τ-analogues.

In [277, 278, 282, 284], an example on event structures with silent actions has been considered demonstrating an independence of ST- and history preserving τ-bisimulation equivalences.

1.3.1 Basic τ-equivalences

The following basic notions of τ-equivalences are known from the literature.

- τ-*trace equivalences* (they respect only protocols of behavior of systems).

  - Interleaving τ-trace equivalence ($\equiv_{i}^{\tau}$) has been introduced in [236] on Petri nets. The definition on transition systems is in [145].
  - Step τ-trace equivalence ($\equiv_{s}^{\tau}$) has been defined in [236] on Petri nets.
  - Partial word τ-trace equivalence ($\equiv_{pw}^{\tau}$) has been proposed in [279] on Petri nets.
  - Pomset τ-trace equivalence ($\equiv_{pom}^{\tau}$) has been defined in [236] on Petri nets.

- *Usual τ-bisimulation equivalences* (they respect branching structure of behavior of systems).

  - Interleaving τ-bisimulation equivalence ($\underleftrightarrow{}_{i}^{\tau}$) has been introduced in [194] on transition systems and in [236, 241] on Petri nets. The definition for transition systems is also in [145].

- Step τ-bisimulation equivalence ($\underleftrightarrow{}_s^\tau$) has been defined in [236] on Petri nets.
- Partial word τ-bisimulation equivalence ($\underleftrightarrow{}_{pw}^\tau$) has been proposed in [278] on Petri nets.
- Pomset τ-bisimulation equivalence ($\underleftrightarrow{}_{pom}^\tau$) has been defined in [236] on Petri nets.

- *ST-τ-bisimulation equivalences* (they respect the duration or maximality of events in behavior of systems).

  - Interleaving ST-τ-bisimulation equivalence ($\underleftrightarrow{}_{iST}^\tau$) has been introduced in [278] on Petri nets.
  - Partial word ST-τ-bisimulation equivalence ($\underleftrightarrow{}_{pwST}^\tau$) [278] on Petri nets.
  - Pomset ST-τ-bisimulation equivalence ($\underleftrightarrow{}_{pomST}^\tau$) has [278] on Petri nets.

- *History preserving τ-bisimulation equivalences* (they respect the "past" or "history" of behavior of systems).

  - Pomset history preserving τ-bisimulation equivalence ($\underleftrightarrow{}_{pomh}^\tau$) has been defined in [113, 114, 115] on Petri nets.

- *History preserving ST-τ-bisimulation equivalences* (they respect the "history" and the duration or maximality of events in behavior of systems).

  - Pomset history preserving ST-τ-bisimulation equivalence ($\underleftrightarrow{}_{pomhST}^\tau$) has been proposed in [113, 114, 115] on Petri nets.

- *Usual branching τ-bisimulation equivalences* (they respect branching structure of behavior of systems taking a special care for silent actions).

  - Interleaving branching τ-bisimulation equivalence ($\underleftrightarrow{}_{ibr}^\tau$) has been defined in [155, 156, 144, 158, 145, 146, 159] on transition systems and in [241] — on Petri nets.

- *History preserving branching τ-bisimulation equivalences* (they respect "history" and branching structure of behavior of systems taking a special care for silent actions).

  - Pomset history preserving branching τ-bisimulation equivalence ($\underleftrightarrow{}_{pomhbr}^\tau$) has been introduced in [113] on Petri nets.

In addition, a number of other τ-equivalences have been defined, and the review of most of them can be found in [145] on transition systems. Among them are the following notions.

Interleaving semi-branching τ-bisimulation equivalence ($\underleftrightarrow{}_{isbr}^\tau$) has been proposed in [155, 156] on transition systems.

Interleaving quasi-branching τ-bisimulation equivalence ($\underleftrightarrow{}_{iqbr}^\tau$) has been defined in [96] on event structures. The definition on transition systems is in [45], and in this paper it is called "branching bisimulation equivalence".

Interleavingη-τ-bisimulation equivalence ($\underleftrightarrow{}_{i\eta}^\tau$) has been introduced in [53] on transition systems.

InterleavingΔ-τ-bisimulation equivalence ($\underleftrightarrow{}_{i\Delta}^\tau$) has been proposed in [195, 287] and in [288] (where it is called "delay bisimulation equivalence") on transition systems.
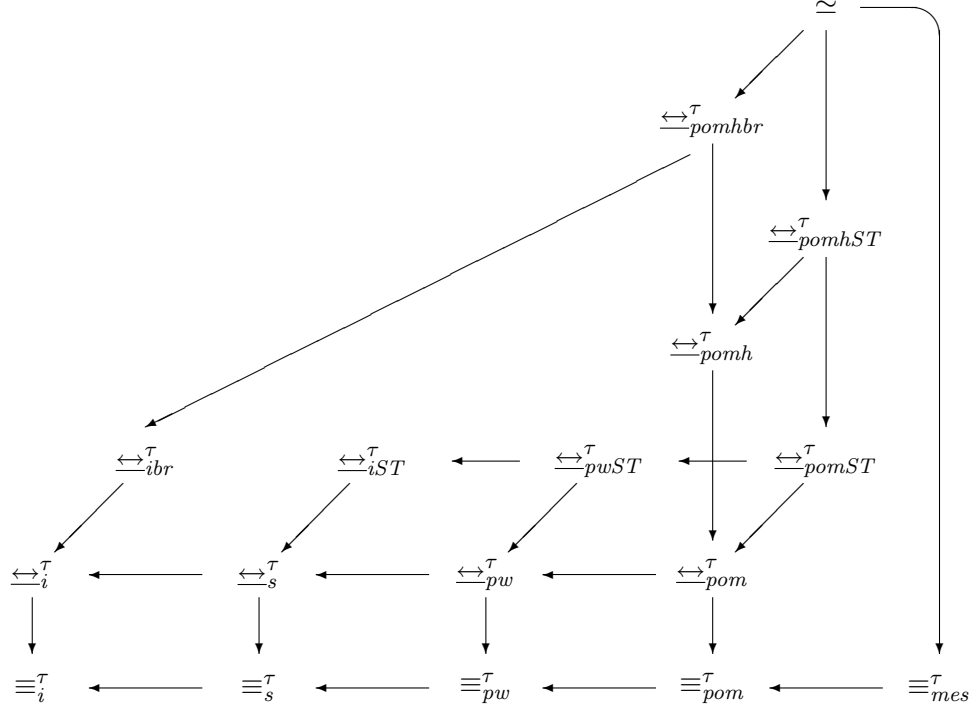
$$\simeq$$

$$\underleftrightarrow{}_{pomhbr}^{\tau}$$

$$\underleftrightarrow{}_{pomhST}^{\tau}$$

$$\underleftrightarrow{}_{pomh}^{\tau}$$

$$\underleftrightarrow{}_{ibr}^{\tau} \qquad \underleftrightarrow{}_{iST}^{\tau} \qquad \underleftrightarrow{}_{pwST}^{\tau} \qquad \underleftrightarrow{}_{pomST}^{\tau}$$

$$\underleftrightarrow{}_{i}^{\tau} \qquad \underleftrightarrow{}_{s}^{\tau} \qquad \underleftrightarrow{}_{pw}^{\tau} \qquad \underleftrightarrow{}_{pom}^{\tau}$$

$$\equiv_{i}^{\tau} \qquad \equiv_{s}^{\tau} \qquad \equiv_{pw}^{\tau} \qquad \equiv_{pom}^{\tau} \qquad \equiv_{mes}^{\tau}$$

Figure 1.14: Interrelations of basic τ-equivalences

In this section, we transfer the mentioned above τ-equivalences into the framework of Petri nets. We also extend the set of basic notions of τ-equivalences by τ-*conflict preserving* (they completely respect conflict): multi event structure τ-conflict preserving equivalence ($\equiv_{mes}^{\tau}$) is introduced.

We compare all the basic τ-equivalences and establish their interrelations.

Comparing basic τ-equivalences

The following theorem which establishes interrelations all the basic τ-equivalences.

Theorem 1.3.1 *Let* $\leftrightarrow, \leftrightarrowtail \in \{\equiv^{\tau}, \underleftrightarrow{}^{\tau}, \simeq\}$, $\star, \star\star \in \{\_, i, s, pw, pom, iST, pwST, pomST, pomh, pomhST, ibr, pomhbr, mes\}$. *For nets* N *and* N' $N \leftrightarrow_{\star} N' \Rightarrow N \leftrightarrowtail_{\star\star} N'$ *iff in the graph in Figure 1.14 there exists a directed path from* $\leftrightarrow_{\star}$ *to* $\leftrightarrowtail_{\star\star}$.

1.3.2 Back-forth τ-equivalences

In this section, we consider back-forth τ-bisimulation equivalences. The following notions of them have been proposed in the literature.

In [210], interleaving back interleaving forth τ-bisimulation equivalence ($\underleftrightarrow{}_{ibif}^{\tau}$) has been defined on transition systems with silent actions. In the paper, it has been demonstrated its coincidence with $\underleftrightarrow{}_{ibr}^{\tau}$.

In [233], on event structures with silent actions, pomset back pomset forth τ-bisimulation equivalence ($\underleftrightarrow{}_{pombpomf}^{\tau}$) has been introduced, and its coincidence with $\underleftrightarrow{}_{pomhbr}^{\tau}$ has been proved.

We complete these back-forth τ-equivalences by 6 new notions: interleaving back step forth ($\underleftrightarrow{}_{ibsf}^{\tau}$), interleaving back partial word forth ($\underleftrightarrow{}_{ibpwf}^{\tau}$), interleaving back pomset forth ($\underleftrightarrow{}_{ibpomf}^{\tau}$),
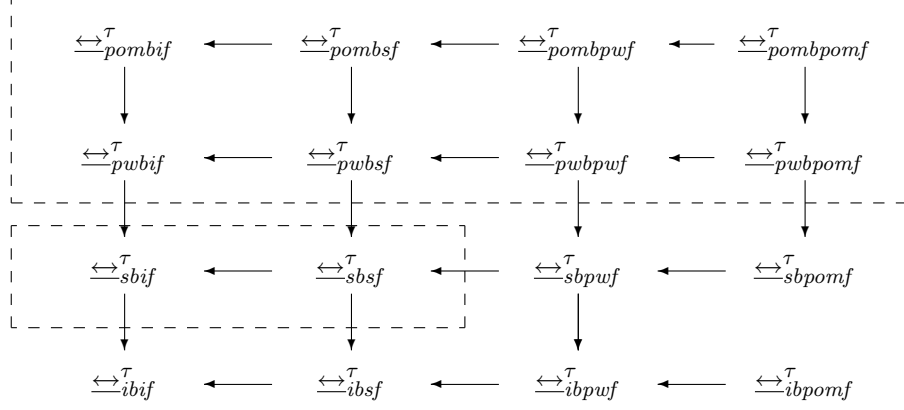
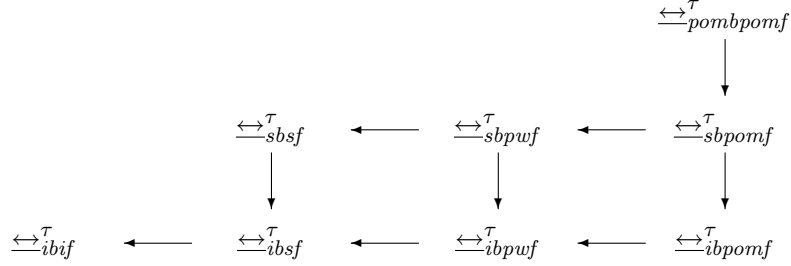Figure 1.15: Merging of back-forth τ-bisimulation equivalences

Figure 1.16: Interrelations of back-forth τ-bisimulation equivalences

step back step forth ($\underline{\leftrightarrow}^\tau_{sbsf}$), step back partial word forth ($\underline{\leftrightarrow}^\tau_{sbpwf}$) and step back pomset forth ($\underline{\leftrightarrow}^\tau_{sbpomf}$) τ-bisimulation equivalences.

We also compare all back-forth τ-equivalences with the set of basic τ-relations.

Comparing back-forth τ-bisimulation equivalences

Let us exam interrelations of back-forth τ-bisimulation equivalences.

Proposition 1.3.1 *Let $\star \in \{i, s, pw, pom\}$. For nets N and N':*

1. *$N \underline{\leftrightarrow}^\tau_{pwb\star f} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^\tau_{pomb\star f} N'$;*

2. *$N \underline{\leftrightarrow}^\tau_{\star bif} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^\tau_{\star b\star f} N'$.*

In Figure 1.15, dashed lines embrace coinciding back-forth τ-bisimulation equivalences.

Hence, interrelations of back-forth τ-bisimulation equivalences may be represented by graph in Figure 1.16.

Interrelations of back-forth τ-bisimulation equivalences with basic τ-equivalences

Let us consider compare back-forth τ-bisimulation equivalences with basic τ-equivalences.
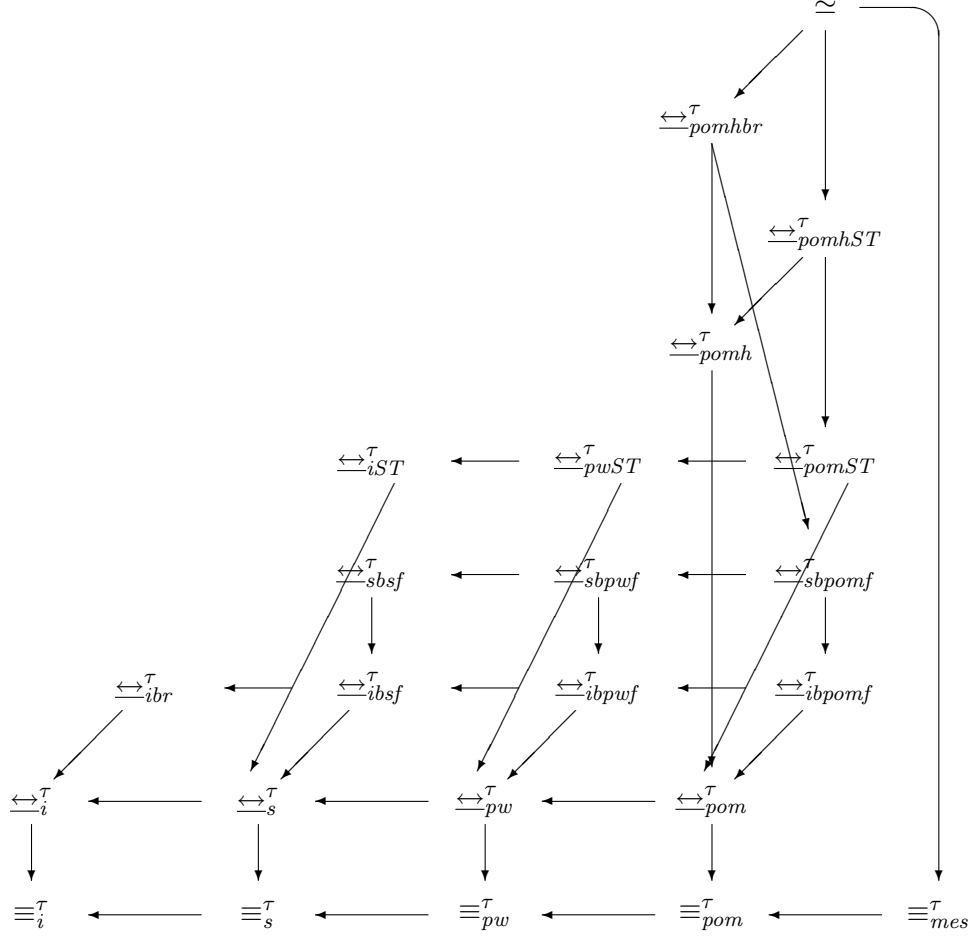
Proposition 1.3.2 *For nets N and N':*

Figure 1.17: Interrelations of back-forth τ-bisimulation equivalences with basic τ-equivalences

1. $N\underline{\leftrightarrow}^{\tau}_{ibif}N' \;\Leftrightarrow\; N\underline{\leftrightarrow}^{\tau}_{ibr}N'$;

2. $N\underline{\leftrightarrow}^{\tau}_{pombpomf}N' \;\Leftrightarrow\; N\underline{\leftrightarrow}^{\tau}_{pomhbr}N'$.

Theorem 1.3.2 *Let* $\leftrightarrow, \leftrightarrow\kern-6pt\leftrightarrow \in \{\equiv^{\tau}, \underline{\leftrightarrow}^{\tau}, \simeq\}$ *and* $\star, \star\star \in \{\_, i, s, pw, pom, iST, pwST, pomST,$ *$pomh, pomhST, ibr, pomhbr, mes, ibsf, ibpwf, ibpomf, sbsf, sbpwf, sbpomf\}$. For nets N and N' $N \leftrightarrow_{\star} N' \;\Rightarrow\; N \leftrightarrow\kern-6pt\leftrightarrow_{\star\star} N'$ iff in the graph in Figure 1.17 there exists a directed path from $\leftrightarrow_{\star}$ to $\leftrightarrow\kern-6pt\leftrightarrow_{\star\star}$.*

Logical characterization

In this section, we demonstrate that several important back-forth (and branching) bisimulation equivalences coincide with that of temporal logics having past modalities. These results provide a logical characterization of bisimulation equivalences (or, dually, an operational characterization of logical ones).

Logic BFL Standard logical equivalence of the logic on Petri nets is denoted by $=_{BFL}$.

A back-forth logic (BFL) has been proposed in [210] in the framework of transition systems for a logical description of the interleaving back interleaving forth bisimulation equivalence.

Theorem 1.3.3 *[210] For two image-finite nets with invisible transitions N and N'*
$$N\underline{\leftrightarrow}^\tau_{ibr}N' \;\Leftrightarrow\; N\underline{\leftrightarrow}^\tau_{ibif}N' \;\Leftrightarrow\; N =_{BFL} N'.$$

Logic $SPBFL$ A pomset back-forth logic with invisible actions ($SPBFL$) has been proposed in [233] in the framework of event structures for a logical description of the pomset back pomset forth bisimulation equivalence.

Standard logical equivalence of the logic on Petri nets is denoted by $=_{SPBFL}$.

Theorem 1.3.4 *[233] For two image-finite nets with invisible transitions N and N'*
$$N\underline{\leftrightarrow}^\tau_{pomhbr}N' \;\Leftrightarrow\; N\underline{\leftrightarrow}^\tau_{pombpomf}N' \;\Leftrightarrow\; N =_{SPBFL} N'.$$

1.3.3   Comparing equivalences with τ-equivalences

In this section, we compare equivalences which do not abstract of silent actions with all the considered τ-equivalences.

Proposition 1.3.3 *Let $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$, $\star \in \{i, s, pw, pom, iST, pwST, pomST, mes, sbsf, sbpwf, sbpomf\}$, $\star\star \in \{s, pw, pom\}$. For nets N and N':*

1. $N \leftrightarrow_\star N' \;\Rightarrow\; N \leftrightarrow^\tau_\star N'$;

2. $N \underline{\leftrightarrow}_i N' \;\Rightarrow\; N \underline{\leftrightarrow}^\tau_{ibr} N'$;

3. $N \underline{\leftrightarrow}_{iST} N' \;\Rightarrow\; N \underline{\leftrightarrow}^\tau_{iSTbr} N'$;

4. $N \underline{\leftrightarrow}_{pomh} N' \;\Rightarrow\; N \underline{\leftrightarrow}^\tau_{pomhSTbr} N'$;

5. $N \underline{\leftrightarrow}_{\star\star} N' \;\Rightarrow\; N \underline{\leftrightarrow}^\tau_{ib\star\star f} N'$.

and all the implications are strict.

1.3.4   Preservation of the τ-equivalences by refinements

In this section, we treat the considered τ-equivalences for preservation by SM-refinements.

In the literature, preservation by SM-refinements has been proved only for $\underline{\leftrightarrow}^\tau_{iST}$ and $\underline{\leftrightarrow}^\tau_{pomhST}$ in [114, 115]. The property is obvious for $\simeq$. The preservation of $\underline{\leftrightarrow}^\tau_{pwST}$ and $\underline{\leftrightarrow}^\tau_{pomST}$ has been demonstrated in [278, 282], but it has been done within framework of event structures, and another refinement operator has been used. The preservation of trace τ-equivalences was not established before.

Thus, our results for $\equiv^\tau_{\star ST}$, $\underline{\leftrightarrow}^\tau_{\star ST}$, $\star \in \{pw, pom\}$ and $\equiv^\tau_{mes}$ are new.

Let us consider the following proposition which demonstrates that some considered in the paper equivalence notions are not preserved by SM-refinements.

Proposition 1.3.4 *Let $\star \in \{i, s\}$, $\star\star \in \{i, s, pw, pom, pomh, ibr, pomhbr, ibsf, ibpwf, ibpomf, sbsf, sbpwf, sbpomf\}$. Then the τ-equivalences $\equiv^\tau_\star$, $\underline{\leftrightarrow}^\tau_{\star\star}$ are not preserved by SM-refinements.*

Theorem 1.3.5 *Let $\leftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ and $\star \in \{\_, i, s, pw, pom, iST, pwST, pomST, pomh, pomhST, ibr, pomhbr, mes, ibsf, ibpwf, ibpomf, sbsf, sbpwf, sbpomf\}$. For nets N, N' s.t. $a \in l_N(T_N) \cap l_{N'}(T_{N'}) \cap Act$ and SM-net D the following holds: $N \leftrightarrow_\star N' \;\Rightarrow\; ref(N, a, D) \leftrightarrow_\star ref(N', a, D)$ iff the equivalence $\leftrightarrow_\star$ is in oval in Figure 1.18.*
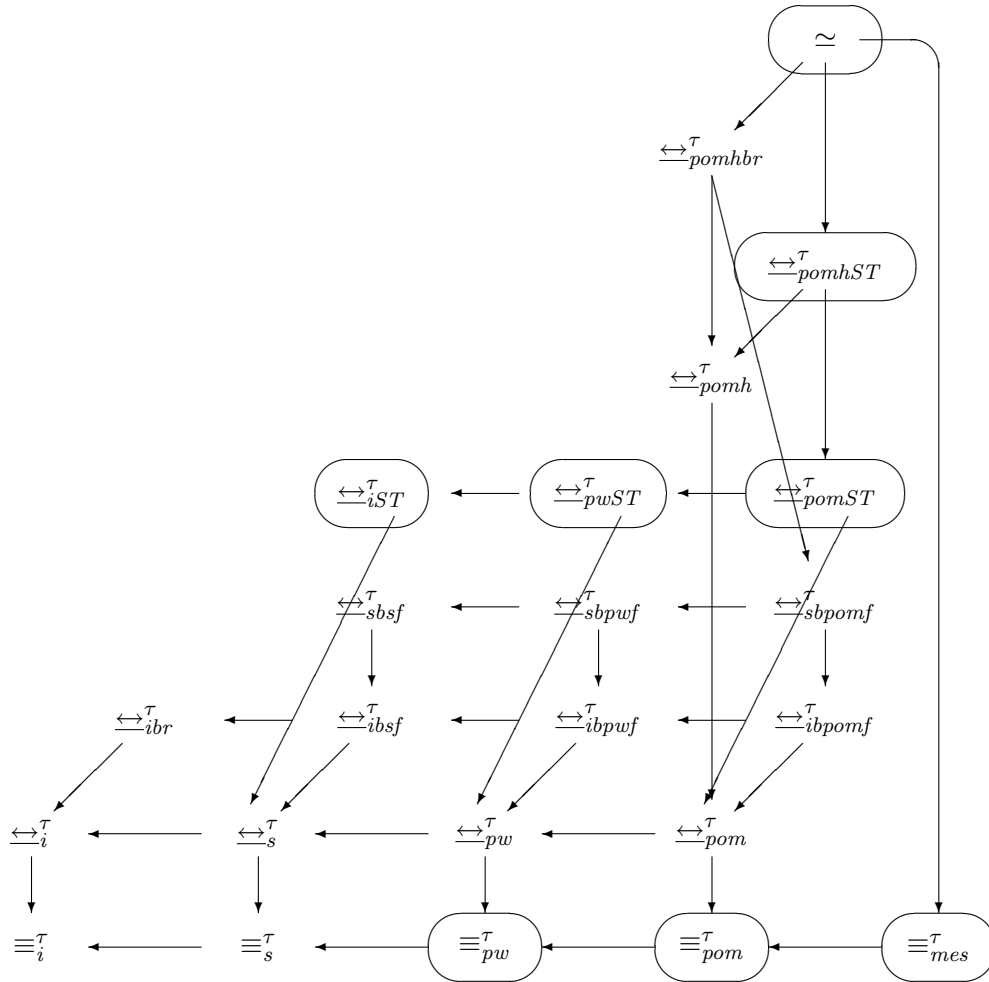
Figure 1.18: The τ-equivalences which are not preserved by SM-refinements

1.3.5 The τ-equivalences on some net subclasses

In this section, we consider the τ-equivalences on nets without silent transitions and sequential nets. Let us note that, in the literature, only $\underline{\leftrightarrow}_i^\tau, \underline{\leftrightarrow}_{pom}^\tau$ and $\underline{\leftrightarrow}_{pomh}^\tau$ have been considered on sequential nets in [39].

The τ-equivalences on nets without silent transitions

Let us consider the τ-equivalences on nets without silent transitions, where no transition is labelled by the action τ.

Proposition 1.3.5 *Let $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$, $\star \in \{i, s, pw, pom, iST, pwST, pomST, mes, sbsf, sbpwf, sbpomf\}$, $\star\star \in \{s, pw, pom\}$. For nets without silent transitions N and N':*

1. $N \leftrightarrow_\star N' \Leftrightarrow N \leftrightarrow_\star^\tau N'$;

2. $N\underline{\leftrightarrow}_{pomh}N' \Leftrightarrow N\underline{\leftrightarrow}_{pomhST}^\tau N'$;

3. $N\underline{\leftrightarrow}_i N' \Leftrightarrow N\underline{\leftrightarrow}_{ibr}^\tau N'$;

4. $N\underline{\leftrightarrow}_{pomh}N' \Leftrightarrow N\underline{\leftrightarrow}_{pomhSTbr}^\tau N'$;

5. $N\underline{\leftrightarrow}_{\star\star}N' \Leftrightarrow N\underline{\leftrightarrow}_{ib\star\star f}^\tau N'$.

In Figure 1.19, dashed lines embrace the τ-equivalences coinciding on nets without silent transitions.

Theorem 1.3.6 *Let $\leftrightarrow, \leftrightarrow\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}, \simeq\}$, $\star, \star\star \in \{\_, i, s, pw, pom, iST, pwST, pomST, pomh, ibr, mes, sbsf, sbpwf, sbpomf\}$. For nets without silent transitions N and N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\leftrightarrow_{\star\star} N'$ iff in the graph in Figure 1.20 there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow\leftrightarrow_{\star\star}$.*

The τ-equivalences on sequential nets

Let us consider the τ-equivalences on sequential nets, where no two transitions can be fired concurrently.

Proposition 1.3.6 *For sequential nets N and N':*

1. $N \equiv_i^\tau N' \Leftrightarrow N \equiv_{pom}^\tau N'$;

2. $N\underline{\leftrightarrow}_i^\tau N' \Leftrightarrow N\underline{\leftrightarrow}_{pomh}^\tau N'$;

3. $N\underline{\leftrightarrow}_{iST}^\tau N' \Leftrightarrow N\underline{\leftrightarrow}_{pomhST}^\tau N'$;

4. $N\underline{\leftrightarrow}_{ibr}^\tau N' \Leftrightarrow N\underline{\leftrightarrow}_{pomhbr}^\tau N'$.

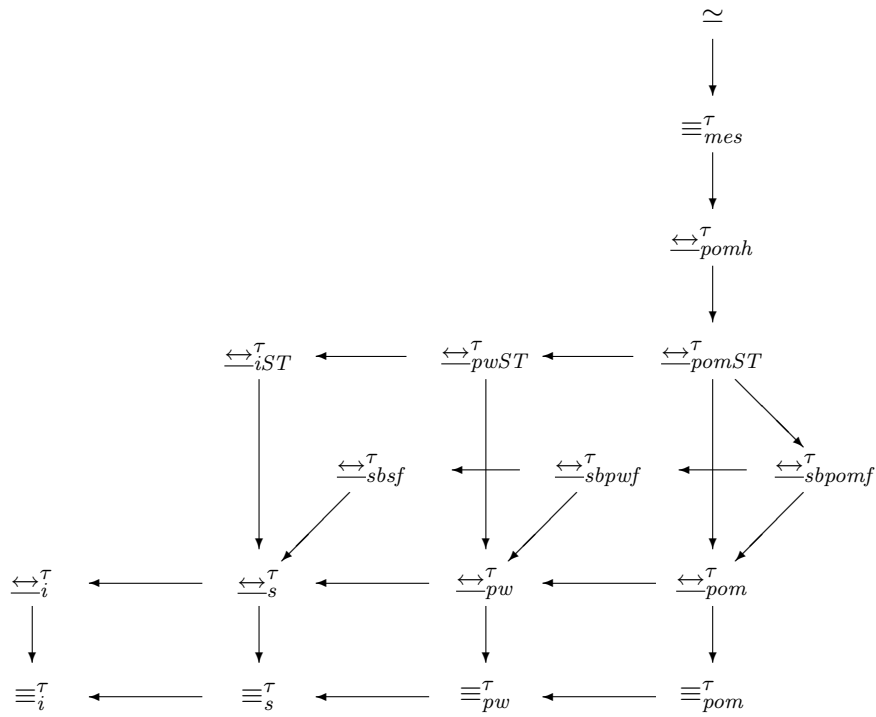In Figure 1.21, dashed lines embrace the τ-equivalences coinciding on sequential nets.

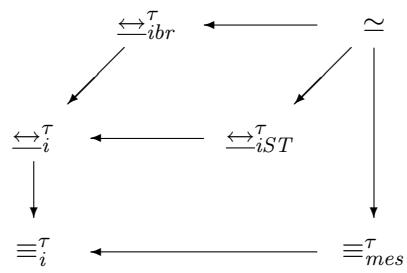Figure 1.19: Merging of the τ-equivalences on nets without silent transitions

$$\underset{\sim}{\simeq}$$

$$\downarrow$$

$$\equiv^{\tau}_{mes}$$

$$\downarrow$$

$$\underleftrightarrow{\tau}_{pomh}$$

$$\downarrow$$

$$\underleftrightarrow{\tau}_{iST} \longleftarrow \underleftrightarrow{\tau}_{pwST} \longleftarrow \underleftrightarrow{\tau}_{pomST}$$

$$\underleftrightarrow{\tau}_{sbsf} \longleftarrow \underleftrightarrow{\tau}_{sbpwf} \longleftarrow \underleftrightarrow{\tau}_{sbpomf}$$

$$\underleftrightarrow{\tau}_{i} \longleftarrow \underleftrightarrow{\tau}_{s} \longleftarrow \underleftrightarrow{\tau}_{pw} \longleftarrow \underleftrightarrow{\tau}_{pom}$$

$$\equiv^{\tau}_{i} \longleftarrow \equiv^{\tau}_{s} \longleftarrow \equiv^{\tau}_{pw} \longleftarrow \equiv^{\tau}_{pom}$$

Figure 1.20: Interrelations of the τ-equivalences on nets without silent transitions

Figure 1.21: Merging of the τ-equivalences on sequential nets

Theorem 1.3.7 *Let $\leftrightarrow, \leftrightarrow\!\!\!\!\rightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$, $\star, \star\!\star \in \{\_, i, iST, ibr, mes\}$. For sequential nets N and N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\!\!\!\!\rightarrow_{\star\star} N'$ iff in the graph in Figure 1.11 there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow\!\!\!\!\rightarrow_{\star\star}$.*

$$\underleftrightarrow{}_{ibr}^{\tau} \longleftarrow \simeq$$

$$\underleftrightarrow{}_{i}^{\tau} \longleftarrow \underleftrightarrow{}_{iST}^{\tau}$$

$$\equiv_{i}^{\tau} \longleftarrow \equiv_{mes}^{\tau}$$

Figure 1.22: Interrelations of the τ-equivalences on sequential nets

Chapter 2

Time Petri nets

Last years, one can see growing interest to real time systems. Hence, there appears a need to embed the notion of time into formal models of systems. A number of specification and analysis methods have been developed which respect time aspects of behavior.

But much less research work has been done to introduce a notion of time into equivalence relations. In several papers [5, 85] by this subject, mainly decidability questions have been discussed.

In the papers, real time systems are modeled by parallel timer processes or time automata containing time measuring elements called clocks. However, concurrency can not be modeled directly by such models.

On the other hand, in the papers [192, 84] time Petri nets have been introduced which are an extension of Petri nets by components respecting time. A time Petri net works in one of two ways: firing of transitions or delaying for some time. Usage of time nets for verification is described, for example, in [295]. In [249], the ideas about history preserving reduction of time nets has been presented.

In this chapter, equivalence notions are investigated on two classes of time Petri nets: that without and with silent transitions.

The main results of this chapter are the following.

- A number of known equivalences have been transferred on time Petri nets from other formal models (for example, time transition systems, parallel timer processes, time automata and algebras of time processes).

- New time (respecting time aspects) and untime (abstracting of time aspects) equivalence relations have been introduced. The purpose was to obtain complete enough set of them to be able to choose available semantics.

- A characterization of time equivalences via regional relations has been done. These relations are based on a concept of regions, which unify infinite number of intermediate states. The states in a region differ each from anther only by value of time counter, and they do not differ in influence to behavior of a time net. Thus, due to the big reduction of states, a checking of time equivalences is simplified.

- A comparison of all the equivalences has been done, and the complete picture of their interrelations has been obtained which demonstrate a differentiating power of them.

- The new operation of time refinement has been introduced, which allows one to change level of structural abstraction while preserving of time delays.

- All the equivalence have been checked for preservation by time refinements to find possible candidates which cam be used during top-down design of time systems.

- Interrelations of equivalences on subclasses of time nets has been investigated. This clarify a nature of the relations and makes easier checking of them.

2.1 Basic definitions

In this section, in the main text of the dissertation, we present basic definitions of time Petri nets, connected with time aspects of their behavior. We use slight modification of model from [192] with time from nonnegative reals and decreasing timers.

2.2 Time Petri nets without silent transitions

Let us consider equivalences for time Petri nets without silent transitions which model systems, all actions of which are considered to be visible for an external observer.

The following basic equivalences for time formal models without invisible actions have been considered in the literature.

- *Time equivalences* (they respect time delays in behavior of systems).

  - Time trace equivalence ($\equiv_t$) has been introduced in [5] on time automata.
  - Time bisimulation equivalence ($\underleftrightarrow{}_t$) has been defined in [85] on parallel timer processes and in [5] — on time automata.

- *Untime equivalences* (they do not respect time delays in behavior of systems).

  - Untime trace equivalence ($\equiv_u$) has been proposed in [5] on time automata.
  - Untime bisimulation equivalence ($\underleftrightarrow{}_u$) had been defined in [5] on time automata.

- *Isomorphism* (i.e. coincidence of time systems up to renaming of their components, $\simeq$) has been introduced in [192].

We also consider region equivalences which partition the states of a TPN into the so-called "regions" [3]. Since we prove that region equivalences coincide with corresponding time ones, a checking of the latter is simplified due to reduction if infinite number of internal states having only different values of time delays. Region bisimulation equivalence ($\underleftrightarrow{}_r$) has been investigated in [85] on parallel timer processes. We introduce new notion of region trace equivalence ($\equiv_r$).

We compare all the relations and prove coincidence of region and trace ones.

Moreover, we introduce new operation of time SM-refinement and check all the equivalences for preservation by it.

At last, we compare all the notions on untime nets, all transitions of which have timers with zero value, and, therefore, can be considered as usual Petri nets without time.

2.2.1 Comparing the equivalences

In this section, we compare all time, untime and region relations. In particular, we prove coincidence of the region equivalence notions with timed ones.

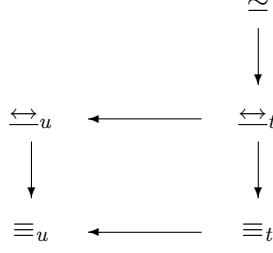**Proposition 2.2.1** *For time nets N and N':*

$$\simeq$$

$$\underleftrightarrow{}_u \longleftarrow \underleftrightarrow{}_t$$

$$\equiv_u \longleftarrow \equiv_t$$

Figure 2.1: Interrelations of the equivalences

$$\boxed{\simeq}$$

$$\underleftrightarrow{}_u \longleftarrow \boxed{\underleftrightarrow{}_t}$$
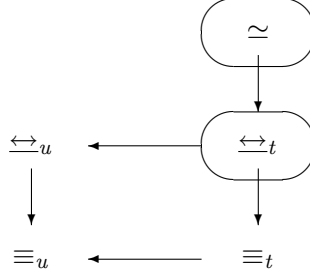
$$\equiv_u \longleftarrow \equiv_t$$

Figure 2.2: Preservation of the equivalences by time SM-refinements

1. $N \equiv_t N' \Leftrightarrow N \equiv_r N'$;

2. $N \underleftrightarrow{}_t N' \Leftrightarrow N \underleftrightarrow{}_r N'$.

The following theorem establishes interrelations of the equivalences.

Theorem 2.2.1 *Let $\leftrightarrow, \leftrightarrow\!\!\!\!\twoheadrightarrow \in \{\equiv, \underleftrightarrow{}, \simeq\}$ and $\star, \star\star \in \{\_, t, u\}$. For time nets N and N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\!\!\!\!\twoheadrightarrow_{\star\star} N'$ iff in the graph in Figure 2.1 there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow\!\!\!\!\twoheadrightarrow_{\star\star}$.*

2.2.2 Preservation of the equivalences by time refinements

In this section, we introduce a notion of time into the known operation of SM-refinement [39] and obtain new operation of time SM-refinement as a result. Time SM-refinement replace transitions of a time Petri net, which are labelled by a particular action, for time SM-net, each "complete" execution of which takes the time coinciding with time delay of a transition to be refined.

Some ideas of time refinement have been considered, for example, in [86] on time process algebras and in [129] — on time Petri nets.

The following theorem demonstrates which equivalences are preserved by time SM-refinements.

Theorem 2.2.2 *Let $\leftrightarrow \in \{\equiv, \underleftrightarrow{}, \simeq\}$ and $\star \in \{\_, t, u\}$. For time nets N, N' s.t. $a \in l_N(T_N) \cap l_{N'}(T_{N'})$ and time SM-net D $N \leftrightarrow_\star N' \Rightarrow tref(N, a, D) \leftrightarrow_\star tref(N', a, D)$ iff the equivalence $\leftrightarrow^\star$ is in oval in Figure 2.2.*

2.2.3 The equivalences on untime Petri nets

In this section, we consider the equivalences on untime nets which have zero delays of transitions, i.e. the transitions fire immediately.

$$\simeq$$

$$\underleftrightarrow{}_u \longleftarrow \underleftrightarrow{}_t$$

$$\equiv_u \longleftarrow \equiv_t$$
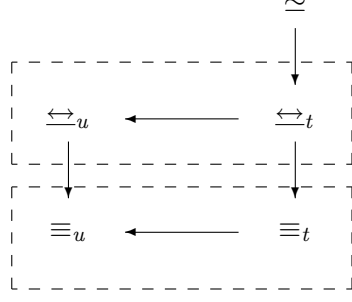
Figure 2.3: Merging of the equivalences on untime nets

$$\simeq$$
$$\downarrow$$
$$\underleftrightarrow{}_u$$
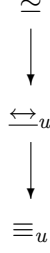$$\downarrow$$
$$\equiv_u$$

Figure 2.4: Interrelations of the equivalences on untime nets

Proposition 2.2.2 *Let $\leftrightarrow \in \{\equiv, \underleftrightarrow{}\}$. For untime nets N and N' $N \leftrightarrow_u N' \Leftrightarrow N \leftrightarrow_t N'$.*

In Figure 2.3, dashed lines embrace the τ-equivalences coinciding on untime nets.

Theorem 2.2.3 *Let $\leftrightarrow, \leftrightarrow\!\!\!\rightarrow \in \{\equiv_u, \underleftrightarrow{}_u, \simeq\}$. For untime nets N and N' $N \leftrightarrow N' \Rightarrow N \leftrightarrow\!\!\!\rightarrow N'$ iff in the graph in Figure 2.4 there exists a directed path from $\leftrightarrow$ to $\leftrightarrow\!\!\!\rightarrow$.*

2.3 Time Petri nets with silent transitions

Let us consider equivalences for time Petri nets with silent transitions which model systems, some actions of which are considered to be invisible for an external observer.

The following basic equivalences for time formal models with invisible actions have been considered in the literature.

- *Time τ-equivalences* (they respect time delays in behavior of systems and abstract from invisible actions).

  - Time τ-bisimulation equivalence ($\underleftrightarrow{}_t^{\tau}$) has been defined in [85] on parallel timer processes.

- *Isomorphism* (i.e. coincidence of time systems up to renaming of their components, $\simeq$) has been introduced in [192].

We introduce the following new relations: time τ-trace equivalence ($\equiv_t^{\tau}$), untime τ-trace equivalence ($\equiv_u^{\tau}$) and untime τ-bisimulation equivalence ($\underleftrightarrow{}_u^{\tau}$).
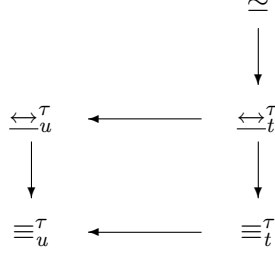
Figure 2.5: Interrelations of the τ-equivalences
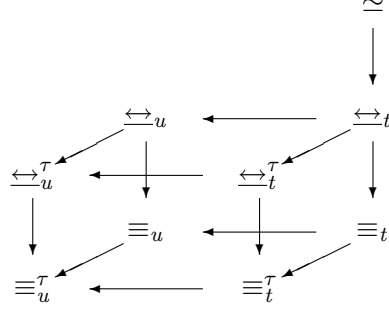
Figure 2.6: Interrelations of equivalences and τ-equivalences

We also consider region τ-equivalences. Region bisimulation τ-equivalence ($\underline{\leftrightarrow}_r^\tau$) has been investigated in [85] on parallel timer processes. We introduce new notion of region τ-trace equivalence ($\equiv_r^\tau$).

We compare all the τ-relations and prove coincidence of region and trace ones.

Moreover, we check all the τ-equivalences for preservation by time SM-refinement operation. At last, we compare all the τ-notions on untime nets.

2.3.1 Comparing the τ-equivalences

In this section, we compare all time, untime and region τ-relations. In particular, we prove coincidence of the region τ-equivalence notions with timed ones.

Proposition 2.3.1 *For time nets with silent transitions N and N':*

1. $N \equiv_t^\tau N' \Leftrightarrow N \equiv_r^\tau N'$;

2. $N \underline{\leftrightarrow}_t^\tau N' \Leftrightarrow N \underline{\leftrightarrow}_r^\tau N'$.

The following theorem establishes interrelations of the equivalences.

Theorem 2.3.1 *Let $\leftrightarrow, \leftrightarrow\in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ and $\star, \star\star \in \{., t, u\}$. For time nets with silent transitions N and N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow_{\star\star} N'$ iff in the graph in Figure 2.5 there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow_{\star\star}$.*
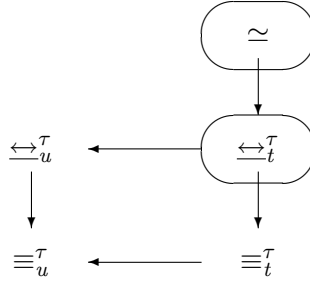
Figure 2.7: Preservation of the τ-equivalences by time SM-refinements

2.3.2 Comparing equivalences and τ-equivalences

Theorem 2.3.2 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\leftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star, \star\star \in \{\_, t, u\}$. *For time nets with silent transitions* N *and* N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\!\!\!\leftrightarrow_{\star\star} N'$ *iff in the graph in Figure 2.6 there exists a directed path from* $\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\leftrightarrow_{\star\star}$.

2.3.3 Preservation of the τ-equivalences by time refinements

In this section, we check τ-equivalences for preservation by time SM-refinement.

The following theorem demonstrates which equivalences are preserved by time SM-refinements.

Theorem 2.3.3 *Let* $\leftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star \in \{\_, t, u\}$. *For time nets with silent transitions* N, N' *s.t.* $a \in l_N(T_N) \cap l_{N'}(T_{N'}) \cap Act$ *and time SM-net* D $N \leftrightarrow_\star N' \Rightarrow tref(N, a, D) \leftrightarrow_\star tref(N', a, D)$ *iff the equivalence* $\leftrightarrow^\star$ *is in oval in Figure 2.7.*

2.3.4 The τ-equivalences on subclasses of time Petri nets with silent transitions

The τ-equivalences on time Petri nets without silent transitions

Let us consider how τ-equivalences are related in case there are no silent transitions.

Proposition 2.3.2 *Let* $\leftrightarrow \in \{\equiv_t, \equiv_u, \underline{\leftrightarrow}_t, \underline{\leftrightarrow}_u\}$. *For time Petri nets without silent transitions* N *and* N' $N \leftrightarrow^\tau N' \Leftrightarrow N \leftrightarrow N'$.

In Figure 2.8, dashed lines embrace the τ-equivalences coinciding on untime nets.

Theorem 2.3.4 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\leftrightarrow \in \{\equiv_t^\tau, \equiv_u^\tau, \underline{\leftrightarrow}_t^\tau, \underline{\leftrightarrow}_u^\tau, \simeq\}$. *For time nets without silent transitions* N *and* N' $N \leftrightarrow N' \Rightarrow N \leftrightarrow\!\!\!\leftrightarrow N'$ *iff in the graph in Figure 2.5 there exists a directed path from* $\leftrightarrow$ *to* $\leftrightarrow\!\!\!\leftrightarrow$.

The τ-equivalences on untime Petri nets with silent transitions

In this section, we consider the τ-equivalences on untime nets with silent transitions which have zero delays of transitions, i.e. they fire immediately.

Proposition 2.3.3 *Let* $\leftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau\}$. *For untime nets with silent transitions* N *and* N' $N \leftrightarrow_u N' \Leftrightarrow N \leftrightarrow_t N'$.
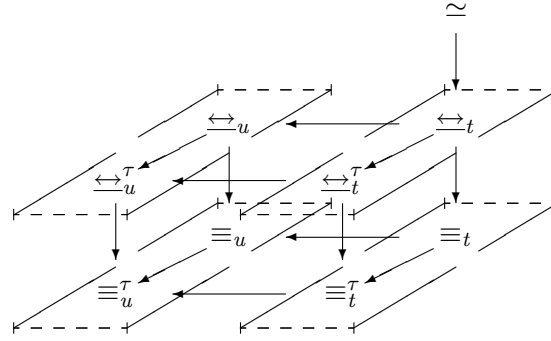
43

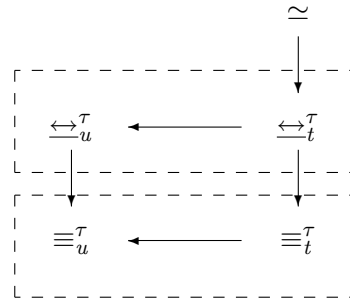Figure 2.8: Merging of the τ-equivalences on time nets without silent transitions

Figure 2.9: Merging of the τ-equivalences on untime nets with silent transitions

In Figure 2.9, dashed lines embrace the τ-equivalences coinciding on untime nets with silent transitions.

Theorem 2.3.5 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\leftrightarrow \in \{\equiv_u^\tau, \underline{\leftrightarrow}_u^\tau, \simeq\}$. *For untime nets with silent transitions N and N' $N \leftrightarrow N'$ $\Rightarrow$ $N \leftrightarrow\!\!\!\leftrightarrow N'$ iff in the graph in Figure 2.10 there exists a directed path from $\leftrightarrow$ to $\leftrightarrow\!\!\!\leftrightarrow$.*
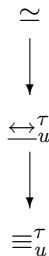
Figure 2.10: Interrelations of the τ-equivalences on untime nets with silent transitions

Chapter 3

Process algebras

For specification of concurrent systems and processes and investigation of their behavioral properties a number of formal models have been proposed. Among these models, algebraic calculi hold a special place. In such algebras, a process is specified by an algebraic formula, and verification of the process properties is accomplished by means of equivalences, axioms and inference rules. All the calculi, considered below, have a common kernel. They construct a process from atomic actions (some of them can communicate) with precedence, parallelism and nondeterminism operations.

1. CCS [193, 194, 199, 200, 74], CSP [167, 74] and its extension [75], TCSP [58, 168, 216, 217], PA [25], BPA [63] and its extension [19, 64, 78] — are the most known algebraic calculi for specification of concurrent processes on the basis of interleaving semantics, where concurrency is modeled by sequential nondeterminism.

2. SCCS [196, 197, 198], ACP [59, 60, 61] and its extensions [18, 23, 24, 25, 69, 53, 54, 55, 56, 62, 140, 76, 77, 20, 21, 22, 128], CCSP [218], PBC [36, 35, 47, 48, 49, 50, 51, 52, 65, 66, 67, 68, 70, 71, 72, 116, 117, 118, 125, 177, 178, 44, 188, 206, 37, 172] have been proposed for axiomatization of a group of concurrency models based on step semantics, where concurrent execution of two processes is simulated by interleaving of multisets of their atomic actions. The set of operations has been extended (comparing with CCS and PA) by new operator for simultaneous occurrence of two actions.

3. Algebraic calculi (for example, algebras from [27, 28]) for a group of models specifying *true concurrency* and based on pomset semantics. Among such models, most well-known are Petri nets [228], event structures [211], pomsets [240], occurrence nets [229], and A-nets [173]. A causal dependence relation over actions imposes the precedence relation, i.e. partial ordering. Hence, two actions are parallel if they are causally independent.

Interleaving calculi are more suitable in technical staff, whereas algebras based on step and pomset semantics have more natural specification of concurrency. By now, all the three approaches coexist, and explanation of this can be found in [41, 42]. New semantics for the known calculi are also proposed. For example, pomset semantics have been defined for CCS [29, 123], CSP [242], SCCS [28], PBC [186], semantics based on Petri nets — for CCS [119, 121, 148, 149, 31], PBC [65, 66, 67, 68], and on the basis of event structures — for CCS [290, 122, 30, 31]. In the same time, different algebraic operations on Petri nets are introduced to impose on nets a modularity which is typical for process algebras [13, 9, 10, 11].

Algebras used for processes specification can be divided by two groups: *"descriptive"* and *"analytical"* ones.

In descriptive algebras like AFP_0, AFP_0^m considered in [91, 98], a process specification provide a description of structural properties of systems.

The calculi $AFP_1, AFP_1^\lambda, AFP_2, AFP_2^\odot, SAFP_1, SAFP_2$ and $SAFP_2$ with λ-actions, considered in [89, 90, 91, 98, 92, 93, 100, 101, 102, 174, 175, 268], have been developed on the basis of "algebra of regular nets" [99, 183, 184]. They are in the third group of algebraic calculi. These algebras combine mechanisms as for specification of concurrent nondeterministic processes as for investigation of their behavioral properties.

The main results of this chapter are the following.

- New algebra of labelled nondeterministic concurrent processes $AFLP_2$ has been introduced. It is an extension of the known calculi AFP_2 [91] by labelling of formulas with action symbols, which provides a possibility to specify and analyze much wider class of processes.

- Net equivalences have been investigated on A-nets and weakly labelled A-nets corresponding to formulas of AFP_2 and $AFLP_2$. The purpose was to make easier comparing the relations with algebraic equivalences and their transferral from nets to algebraic formulas.

- An operational characterization of the equivalence w.r.t. denotational semantics of $AFLP_2$ has been proposed which may be used to compare semantical equivalence with behavioral relations.

- A sound and complete axiomatization of semantical equivalences has been done which makes weaker comparing different formula specifications of processes.

- Algebraic equivalences have been transferred to nets. Net and algebraic equivalence notions have been compared, and it clarified the nature of the latter.

- Net equivalences have been transferred to formulas of $AFLP_2$. A soundness of algebraic analogues with the initial net relations has been proved. It has provided one with a possibility to change net by algebraic specifications and back while preserving behavior. One can also combine advantages of nets and algebras.

- Algebraic analogues of the net equivalences have been checked for compositionality via structural operations.

- A term rewrite system RWS_2 has been proposed to make checking semantic equivalences of AFP_2 and $AFLP_2$ automatic. Its confluence in the case of termination has been proved.

- On the basis of RWS_2, a program in C programming language has been written which implement checking of formula equivalences for aforementioned algebras.

3.1   Algebra of finite processes AFP_2

Algebra AFP_2 considered in this section is based on semantics of posets with non-actions and deadlocked actions which help to preserve information about nondeterminism. Synchronization of actions in AFP_2 is by name. This means that several equally named actions possibly from different parts of a formula of the calculus are synchronized. One event is considered to correspond all these actions.

In [91], it was demonstrated that by means of formulas of AFP_2 one can analyze behavior of A-nets. In this section, the proposed before equivalences are examined on this net subclass. Equivalences on formulas AFP_2 are transferred into A-nets, and their interrelations with the net equivalences are investigated.

A method of automatized checking of semantical formula equivalences based on term rewrite system RWS_2 is proposed. We prove confluence of the system in case of termination. The method is implemented as a program in C language.

3.1.1 Syntax

Let $\alpha = \{a, b, c, \ldots\}$ be an alphabet of symbols of *actions*, $\bar{\alpha} = \{\bar{e}, \bar{f}, \ldots\}$ be symbols of *non-actions* and $\Delta_\alpha = \{\delta_e, \delta_f, \ldots\}$ be symbols of *deadlocked actions*. Let us denote $\hat{\alpha} = \alpha \cup \bar{\alpha} \cup \Delta_\alpha$. Symbols of $\hat{\alpha}$ are combined into formulas by operations ; (*precedence*), $\triangledown$ (*exclusive or, alternative*), $\|$ (*concurrency*), $\vee$ (*disjunction, union*), $\rrbracket$ (*"not occur"*), $\tilde{\rrbracket}$ (*"mistaken not occur"*).

A *formula* of AFP_2 in a basis $\hat{\alpha}$ is defined by the following production system.

$$E ::= \ a \mid \bar{a} \mid \delta_a \mid \rrbracket a \mid \tilde{\rrbracket} P \mid P; Q \mid P\|Q \mid P \triangledown Q \mid P \vee Q$$

Here $a \in \alpha$, $\bar{a} \in \bar{\alpha}$, $\delta_a \in \Delta_\alpha$ are *elementary formulas*. We denote by $\mathbf{AFP_2}$ *a set of all formulas of AFP_2*.

Let P and P' be formulas of AFP_2. P and P' are *isomorphic*, notation $P \simeq P'$, if these formulas coincide up to associativity rules w.r.t. ;, $\|, \vee, \triangledown$ and commutativity rules w.r.t. $\|, \vee, \triangledown$.

3.1.2 Semantics

Denotational semantics

A semantics of a process specified by a formula P of AFP_2, notation $\mathcal{D}_{F2}[P]$, is a set of partial orders considered in [91, 92], with ordering in accordance to the dependence relation.

We introduce two equivalences w.r.t denotational semantics. The first is usual one ($=_{\mathcal{D}_{F2}}$), which means coincidence of pomset sets corresponding to the formulas to be compared. The second is observable one ($=_{\mathcal{D}_{F2}^+}$), which resembles the previous equivalence with exception it abstracts from non-actions and deadlocked actions.

Axiomatization

In accordance to the equivalence w.r.t. denotational semantics, an axiom system Θ_{F2} is proposed.

In the following equalities, P, Q, $R \in \mathbf{AFP_2}$, $a \in \alpha$, $\bar{a} \in \bar{\alpha}$, $\delta_a \in \Delta_\alpha$.

1. Associativity

    1.1 $P\|(Q\|R) = (P\|Q)\|R$
    1.2 $P \triangledown (Q \triangledown R) = (P \triangledown Q) \triangledown R$
    1.3 $P \vee (Q \vee R) = (P \vee Q) \vee R$
    1.4 $P; (Q; R) = (P; Q); R$

2. Commutativity

    2.1 $P\|Q = Q\|P$

2.2 $P \bigtriangledown Q = Q \bigtriangledown P$

2.3 $P \vee Q = Q \vee P$

3. Distributivity

   3.1 $(P\|Q); R = (P; R)\|(Q; R)$

   3.2 $P; (Q\|R) = (P; Q)\|(P; R)$

   3.3 $(P \vee Q); R = (P; R) \vee (Q; R)$

   3.4 $P; (Q \vee R) = (P; Q) \vee (P; R)$

   3.5 $(P \vee Q)\|R = (P\|R) \vee (Q\|R)$

   3.6 $P \bigtriangledown (Q\|R) = (P \bigtriangledown Q)\|(P \bigtriangledown R)$

4. Axioms for $\bigtriangledown$ and $\rceil\!\rceil$

   4.1 $P \bigtriangledown Q = (P\|(\rceil\!\rceil Q)) \vee ((\rceil\!\rceil P)\|Q)$

   4.2 $\rceil\!\rceil(P\|Q) = (\rceil\!\rceil P)\|(\rceil\!\rceil Q)$

   4.3 $\rceil\!\rceil(P \vee Q) = (\rceil\!\rceil P) \vee (\rceil\!\rceil Q)$

   4.4 $\rceil\!\rceil(P; Q) = (\rceil\!\rceil P)\|(\rceil\!\rceil Q)$

   4.5 $\rceil\!\rceil a = \bar{a}$

   4.6 $\rceil\!\rceil \bar{a} = \bar{a}$

   4.7 $\rceil\!\rceil \delta_a = \bar{a}$

5. Structural properties

   5.1 $\bar{a}; P = \bar{a}\|P$

   5.2 $P; \bar{a} = P\|\bar{a}$

   5.3 $P\|(P; Q) = (P; Q)$

   5.4 $Q\|(P; Q) = (P; Q)$

   5.5 $P; Q; R = (P; Q)\|(Q; R)$

   5.6 $(P; Q)\|(Q; R) = (P; Q)\|(Q; R)\|(P; R)$

   5.7 $P\|P = P$

   5.8 $P \vee P = P$

   5.9 $P \vee Q = P$ or $Q \triangleleft P$ (a concept of strict prefix $\triangleleft$ for formulas is defined in the main text of the dissertation)

6. Axioms for deadlocked events and $\tilde{\rceil\!\rceil}$

   6.1 $a\|\bar{a} = \delta_a$

   6.2 $a; a = \delta_a$

   6.3 $a\|\delta_a = \delta_a$

   6.4 $\delta_a; P = \delta_a\|(\tilde{\rceil\!\rceil} P)$

   6.5 $P; \delta_a = P\|\delta_a$

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6.6 $\delta_a \| (\P P) = \delta_a \| (\tilde{\P} P)$

6.7 $\tilde{\P} a = \delta_a$

6.8 $\tilde{\P} \bar{a} = \delta_a$

6.9 $\tilde{\P} \delta_a = \delta_a$

6.10 $\tilde{\P} (P \| Q) = (\tilde{\P} P) \| (\tilde{\P} Q)$

6.11 $\tilde{\P} (P; Q) = (\tilde{\P} P) \| (\tilde{\P} Q)$

6.12 $\tilde{\P} (P \vee Q) = (\tilde{\P} P) \vee (\tilde{\P} Q)$

The axiom system Θ_{FL2} is *sound* for $=_{\mathcal{D}_{FL2}}$, i.e. if $P = P'$ is an axiom of Θ_{FL2}, then $P =_{\mathcal{D}_{FL2}} P'$. The proof consists in determining the semantics of P and P' and comparing them. It can be done directly by the definitions of operations over lposets.

In order to prove that Θ_{FL2} is *complete* for $=_{\mathcal{D}_{FL2}}$, we introduce a canonical form of $AFLP_2$-formula.

Canonical form of formulas

In this section, a canonical form of AFP_2-formulas is introduced, which is disjunctive normal form whose elementary members are symbols from $\hat{\alpha}$ or elementary precedences (of two actions). As conjunction, we use $\|$, and disjunction is $\vee$.

The notation $P =_{\Theta_{F2}} P'$ means that the equality of P and P' can be proved using Θ_{F2}.

The following theorems prove completeness of Θ_{F2}.

Theorem 3.1.1 *[91, 92] Any formula of AFP_2 can be reduced to the unique (up to isomorphism) canonical form.*

Theorem 3.1.2 *[91, 92] For any two formulas P and P' of AFP_2 the following holds: $P =_{\mathcal{D}_{F2}} P' \Leftrightarrow P =_{\Theta_{F2}} P'$.*

Thus, one can easily check whether two formulas of AFP_2 P and P' are equivalent. For this, it is enough to reduce them to canonical forms Q and Q' and check by coincidence up to isomorphism.

3.1.3 Equivalences on A-nets

In [183, 184], an algebra of regular nets has been introduced. Later, on the basis of this calculus, an algebra AFP_0 has been proposed s.t. its formulas correspond to finite A-nets [173, 185]. In [183, 184], it has been proved that any finite A-net can be specified by a formula of the algebra using "regularization" algorithm. In [91], a mapping Ψ has been defined from the set of all formulas of AFP_0 into that of AFP_2. This mapping has a property stating that the set of posets of the net specified by a formula P of AFP_0, coincide with the set of posets of nondeterministic process specified by the formula $\Psi(P)$ of AFP_2. It allows one not to differentiate between formulas P and $\Psi(P)$ in pomset semantics. Hence, given the A-net specified by a formula P of AFP_0, one can analyze its properties and behavior by means ot the same formula P of AFP_2.

Proposition 3.1.1 *For A-nets N and N':*

1. $N \equiv_i N' \Leftrightarrow N \equiv_{mes} N'$;

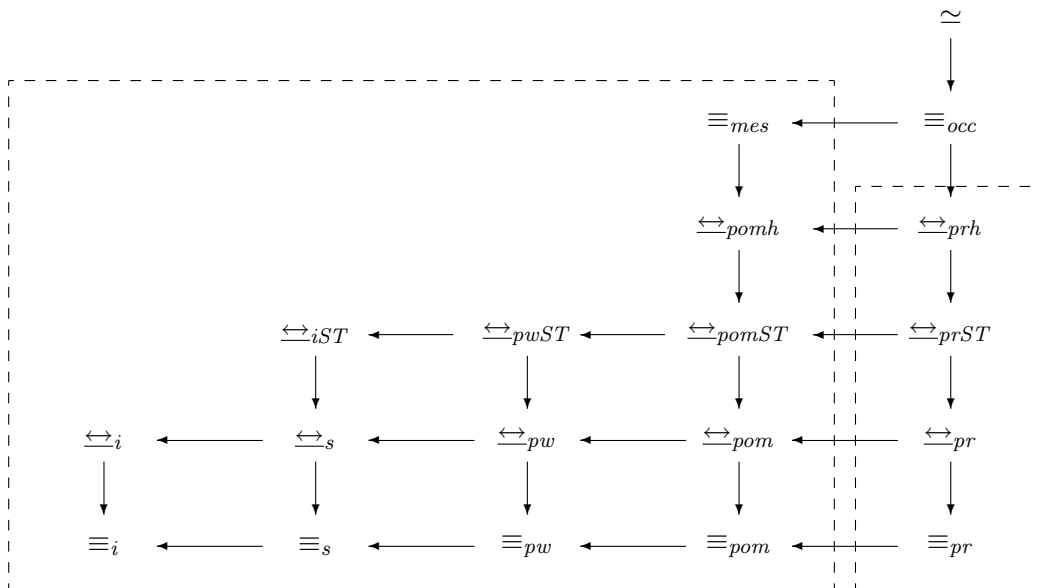*2. $N \equiv_{pr} N' \Leftrightarrow N \underleftrightarrow{}_{prh} N'$.*

$$\simeq$$

$$\downarrow$$

$\equiv_{mes} \longleftarrow\!\!\!\!\mid\!\!\!\!- \equiv_{occ}$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$\underleftrightarrow{}_{pomh} \longleftarrow\!\!\!\!\mid\!\!\!\!- \underleftrightarrow{}_{prh}$

$\underleftrightarrow{}_{iST} \longleftarrow \underleftrightarrow{}_{pwST} \longleftarrow \underleftrightarrow{}_{pomST} \longleftarrow\!\!\!\!\mid\!\!\!\!- \underleftrightarrow{}_{prST}$

$\underleftrightarrow{}_{i} \longleftarrow \underleftrightarrow{}_{s} \longleftarrow \underleftrightarrow{}_{pw} \longleftarrow \underleftrightarrow{}_{pom} \longleftarrow\!\!\!\!\mid\!\!\!\!- \underleftrightarrow{}_{pr}$

$\equiv_{i} \longleftarrow \equiv_{s} \longleftarrow \equiv_{pw} \longleftarrow \equiv_{pom} \longleftarrow\!\!\!\!\mid\!\!\!\!- \equiv_{pr}$

Figure 3.1: Merging of the basic equivalences on A-nets

$$\simeq$$

$$\downarrow$$

$$\equiv_{occ}$$

$$\downarrow$$

$\equiv_{i} \longleftarrow \equiv_{pr}$
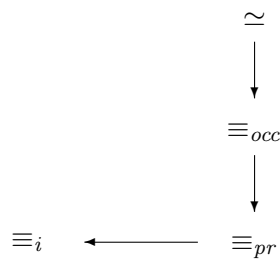
Figure 3.2: Interrelations of the basic equivalences on A-nets

$$=_{\mathcal{D}_{F2}} \longleftarrow \quad \simeq$$

$$\Big\downarrow \qquad\qquad \Big\downarrow$$

$$\equiv_{occ}$$

$$\Big\downarrow \qquad\qquad \Big\downarrow$$

$$=_{\mathcal{D}_{F2}^+} \;=\; \equiv_i \quad \longleftarrow \quad \equiv_{pr}$$
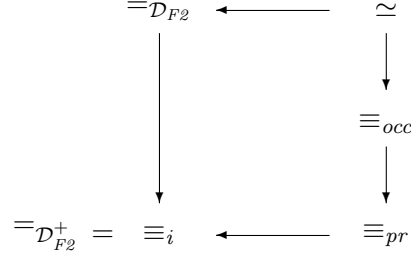
Figure 3.3: Interrelations of the basic net and algebraic equivalences

In Figure 3.1, dashed lines embrace the basic equivalences coinciding on A-nets.

Theorem 3.1.3 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\!\leftrightarrow\, \in \{\equiv, \simeq\}$, $\star, \star\star \in \{\_, i, pr, occ\}$. *For A-nets* N *and* N' $N \leftrightarrow_\star N' \Rightarrow$ $N \leftrightarrow\!\!\!\!\leftrightarrow_{\star\star} N'$ *iff there exists a directed path from* $\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\!\leftrightarrow_{\star\star}$ *in the graph in Figure 3.2.*

3.1.4 Comparing the net and algebraic equivalences

Equivalence notions for AFP_2 can be extended to nets. For this, it is enough to find a formula of AFP_2 for every finite A-net, as it has been described in the previous section.

Let $\leftrightarrow$ be a formula equivalence of AFP_2, and the formulas E and E' correspond to the finite A-nets N and N'. We say that two nets N and N' *are equivalent (w.r.t.* $\leftrightarrow$*)*, notation $N \leftrightarrow N'$, iff the formulas corresponding them are also equivalent, i.e. $E \leftrightarrow E'$.

The following proposition states that on A-nets observed equivalence w.r.t. denotational semantics coincide with interleaving trace one.

Proposition 3.1.2 *For A-nets* N *and* N' $N \equiv_i N' \Leftrightarrow N =_{\mathcal{D}_{F2}^+} N'$.

Theorem 3.1.4 *Let* $\leftrightarrow, \leftrightarrow\!\!\!\!\leftrightarrow\, \in \{\equiv, \simeq, =\}$, $\star, \star\star \in \{\_, i, pr, occ, \mathcal{D}_{F2}^+, \mathcal{D}_{F2}\}$. *For A-nets* N *and* N' $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow\!\!\!\!\leftrightarrow_{\star\star} N'$ *iff there exists a directed path from* $\leftrightarrow_\star$ *to* $\leftrightarrow\!\!\!\!\leftrightarrow_{\star\star}$ *in the graph in Figure 3.3.*

3.1.5 Automatization checking for algebraic equivalences

A process of transformation of a formula of AFP_2 into canonical form with the use of the axiom system Θ_{F2} is not straightforward. It is especially hard if a formula is long and has an intricate structure, as happens very often while formula specification of real life processes. Moreover, during the transformation, the axioms of Θ_{F2} are often applied not only in usual "direction" (i.e. from the left side to the right one) but also in reverse one.

To automatically transform the initial formula to one of isomorphic canonical forms, one have to design a term rewrite system. Application of rules of the system produces such a transformation.

In accordance to these requirements, we develop a term rewrite system RWS_2.

Term rewrite system RWS_2

A *substitution* of subformula P_i of a formula P by another subformula Q, denoted by $[P]_Q^{P_i}$, is the formula $P_1 \circ \ldots P_{i-1} \circ Q \circ P_{i+1} \circ \ldots \circ P_n$, where $P = P_1 \circ \ldots \circ P_{i-1} \circ P_i \circ P_{i+1} \circ \ldots \circ P_n$, $\circ \in \{;, \|, \bigtriangledown, \vee\}$.

In the following rules of RWS_2 P, Q, R denote formulas of AFP_2, a, b, $c \in \alpha$, $\bar{a}$, $\bar{b} \in \bar{\alpha}$, δ_a, $\delta_b \in \Delta_\alpha$. The digits in parentheses are the numbers of equalities of Θ_{F2} which has been used for producing corresponding transition rules.

The notions like "1,2,3-conjunction", "1,2-disjunction" or "normal disjunction" correspond to the particular good structural properties of a formula. They have been presented before definition of canonical form in the initial text of dissertation, and omitted in this survey as too technical. This should not produce any problems with understanding.

1.1. $\circ \in \{;, \|, \vee\} \Rightarrow$
$P \circ (Q \circ R) \to (P \circ Q) \circ R$
(1.1, 1.3, 1.4);

2.1. $(\bullet, \circ) \in \{(\|, ;), (\vee, ;), (\vee, \|)\} \Rightarrow$
$(P \circ Q) \bullet R \to (P \bullet R) \circ (Q \bullet R)$
(3.1, 3.3, 3.5);

2.2. $(\bullet, \circ) \in \{(\|, ;), (\vee, ;), (\vee, \|)\} \Rightarrow$
$P \bullet (Q \circ R) \to (P \bullet Q) \circ (P \bullet R)$
(2.1, 3.2, 3.4, 3.5);

3.1 $P \triangledown Q \to (P\|(\rceil Q)) \vee ((\rceil P)\|Q)$
(4.1);

4.1. $\circ \in \{\|, ;\}, \neg \in \{\rceil, \tilde{\rceil}\} \Rightarrow$
$\neg(P \circ Q) \to (\neg P)\|(\neg Q)$
(4.2, 4.4, 6.10, 6.11);

4.2. $\neg \in \{\rceil, \tilde{\rceil}\} \Rightarrow$
$\neg(P \vee Q) \to (\neg P) \vee (\neg Q)$
(4.3, 6.12);

4.3. $P = a$ or $P = \bar{a}$ or $P = \delta_a \Rightarrow$
$\rceil P \to \bar{a}$
(4.5, 4.6, 4.7);

4.4. $P = a$ or $P = \bar{a}$ or $P = \delta_a \Rightarrow$
$\tilde{\rceil} P \to \delta_a$
(6.7, 6.8, 6.9);

5.1. $P, Q, R \in \hat{\alpha} \Rightarrow$
$(P; Q); R \to ((P; Q)\|(Q; R))\|(P; R)$
(5.5, 5.6);

5.2. $Q \in \hat{\alpha} \Rightarrow$
$\bar{a}; Q \to \bar{a}\|Q$
(5.1);

5.3. $P \in \hat{\alpha} \Rightarrow$
$P; \bar{a} \to P\|\bar{a}$
(5.2);

5.4. $a; a \to \delta_a$
(6.2);

5.5. $Q = b$ or $Q = \bar{b}$ or $Q = \delta_b$ $\Rightarrow$
$\delta_a; Q \to \delta_a \| \delta_b$
(6.4, 6.7, 6.8, 6.9);

5.6. $P \in \hat{\alpha}$ $\Rightarrow$
$P; \delta_a \to P \| \delta_a$
(6.5);

6.1. P is 1-conjunction, $P' = \delta_a$ is a conjunctive member of P $\Rightarrow$
$P \| \bar{b} \to P \| \delta_b$
(1.1, 2.1, 4.5, 6.6, 6.7);

6.2. P is 1-conjunction, $P' = \bar{b}$ is a conjunctive member of P $\Rightarrow$
$P \| \delta_a \to [P]_{\delta_b}^{P'} \| \delta_a$
(1.1, 2.1, 4.5, 6.6, 6.7);

7.1. P is 1,2-conjunction, P' is a conjunctive member of P, $P' = a$ or $P' = b$ $\Rightarrow$
$P \| (a; b) \to [P]_{(a;b)}^{P'}$
(1.1, 2.1, 5.3, 5.4);

7.2. P is 1,2-conjunction, P' is a conjunctive member of P, $P' = (a; b)$ or $P' = (b; a)$ $\Rightarrow$
$P \| a \to P$
(1.1, 2.1, 5.3, 5.4);

7.3. P is 1,2-conjunction, $P' = a$ is a conjunctive member of P, $\diamond \in \{^-, \delta\}$ $\Rightarrow$
$P \| \diamond a \to [P]_{\delta_a}^{P'}$
(1.1, 2.1, 6.1, 6.3);

7.4. P is 1,2-conjunction, P' is a conjunctive member of P, $P' = \bar{a}$ or $P' = \delta_a$ $\Rightarrow$
$P \| a \to [P]_{\delta_a}^{P'}$
(1.1, 2.1, 6.1, 6.3);

7.5. P is 1,2-conjunction, $P' = (a; b)$ is a conjunctive member of P, $\diamond \in \{^-, \delta\}$ $\Rightarrow$
$P \| \diamond a \to [P]_{\delta_b}^{P'} \| \delta_a$
(1.1, 1.4, 2.1, 5.1, 6.1, 6.3, 6.4, 6.7);

7.6. P is 1,2-conjunction, $P' = (b; a)$ is a conjunctive member of P, $\diamond \in \{^-, \delta\}$ $\Rightarrow$
$P \| \diamond a \to [P]_{b}^{P'} \| \delta_a$
(1.1, 2.1, 5.2, 6.1, 6.3, 6.5);

7.7. P is 1,2-conjunction, P' is a conjunctive member of P, $P' = \bar{a}$ or $P' = \delta_a$ $\Rightarrow$
$P \| (a; b) \to [P]_{\delta_a}^{P'} \| \delta_b$
(1.1, 1.4, 2.1, 5.1, 6.1, 6.3, 6.4, 6.7);

7.8. P is 1,2-conjunction, P' is a conjunctive member of P, $P' = \bar{a}$ or $P' = \delta_a$ $\Rightarrow$
$P \| (b; a) \to [P]_{\delta_a}^{P'} \| b$
(1.1, 2.1, 5.2, 6.1, 6.3, 6.5);

7.9. P is 1,2-conjunction, $P' = Q$ is a conjunctive member of P $\Rightarrow$
$P \| Q \to P$
(1.1, 2.1, 5.7);

8.1. P is 1,2,3-conjunction, $P' = (a;b)$ is a conjunctive member of P, in the maximal 1,2,3-conjunction containing P as a conjunctive member, there is no conjunctive member
$P'' = (a;c) \Rightarrow$
$P\|(b;c) \to (P\|(b;c))\|(a;c)$
(1.1, 2.1, 5.6);

8.2. P is 1,2,3-conjunction, $P' = (c;a)$ is a conjunctive member of P, in the maximal 1,2,3-conjunction containing P as a conjunctive member there is no conjunctive member
$P'' = (b;a) \Rightarrow$
$P\|(b;c) \to (P\|(b;c))\|(b;a)$
(1.1, 2.1, 5.6);

9.1. P is 1-disjunction, P' is a disjunctive member of P, $P' \simeq Q \Rightarrow$
$P \vee Q \to P$
(1.1, 1.3, 2.1, 2.3, 5.8);

10.1. P is 1,2-disjunction, Q is a normal conjunction, P' is a disjunctive member of P, $Q \triangleleft P' \Rightarrow$
$P \vee Q \to P$
(1.3, 2.3, 5.9);

10.2. P is 1,2-disjunction, Q is a normal conjunction, P' is a disjunctive member of P, $P' \triangleleft Q \Rightarrow$
$P \vee Q \to [P]_Q^{P'}$
(1.3, 2.3, 5.9).

A proof of confluence of RWS_2

Proposition 3.1.3 *No rule of a RWS_2 can be applied to a formula P of AFP_2 iff P is in canonical form.*

Hence, to check semantical equivalence of two AFP_2-formulas, it is enough to transform them to their canonical forms with the use of RWS_2 and then compare these canonical forms by isomorphism.

The author developed program in C programming language (3000 lines) based on the results of this section. The program automatically transforms any AFP_2 into canonical form. Description of the and examples of its work can be found in Appendices B and C.

3.2 Algebra of finite labelled processes $AFLP_2$

An advantage of considered above algebra AFP_2 is a mechanism of action synchronization by name which is close to the net one. The mechanism allows one to represent processes which cannot be specified by formulas of other calculi (for example, CCS, TCSP, ACP or algebra of event structures from [27, 28]), where different unique events correspond to every appearance of certain action in a formula [91]. In [121], a semantics of CCS is proposed based on Petri nets. Formulas of CCS correspond to C/E-systems (a subclass of Petri nets). It has been demonstrated that the reverse translation (C/E-nets into CCS-formulas) is impossible due to the binarity of synchronization operation in CCS. On the other hand, in Petri nets, a synchronization can be not only binary one, since a transition can have more than two pre-conditions (input places). In the same paper, it is stressed that the synchronization operator should be extended to solve the problem.

At the same time, in the framework of AFP_2, it is impossible to specify a process with two concurrent actions having the same name.

In this section, we introduce an algebra $AFLP_2$ (Algebra of Finite Labelled Processes) on the basis of AFP_2 by imposing the global labelling to event symbols which are combined into formulas. Hence, the formulas of $AFLP_2$ specify *labelled* nondeterministic processes where several different events may be equally labelled,unlike AFP_2-formulas. Thus, using $AFLP_2$, one can specify much wider class of processes than that in AFP_2.

Like AFP_2, calculus $AFLP_2$ is as descriptive as analytical one. Thus, by means of $AFLP_2$, it is possible as specify a process as analyze its properties.

From algebra AFP_2, calculus $AFLP_2$ inherited the mechanism of non-actions and deadlocked actions which allows one to preserve the information about non-determinism.

In $AFLP_2$, denotational semantics is introduced on the basis of labelled posets (lposets) with non-events and deadlocked events. The semantical equivalence of $AFLP_2$-formulas is defined, and sound and complete axiom set corresponding to the equivalence is presented.

The possibility to use the program CANON for automatized checking of semantical equivalence of $AFLP_2$ is mentioned.

Operational semantics of $AFLP_2$ is defined based on transition systems connected with pomsets. In this system, a transformation of a formula in accordance to the rules is interpreted as occurrence of a pomset.

A coincidence ot denotational and operational semantics is proved.

It is demonstrated that by means of $AFLP_2$ one can analyze the behavior of weakly labelled A-nets (i.e. A-nets which may have noninjective labelling function). The net equivalences considered before are treated on this subclass of Petri nets. Semantical equivalences of $AFLP_2$ are transferred into weakly labelled A-nets, and their interrelations with the net equivalences are examined.

Analogues of the net equivalences are introduced on $AFLP_2$-formulas, and their accordance with the original net equivalences is proved.

At last, the fact is established that semantical equivalence of $AFLP_2$ is the only one which is a congruence w.r.t. operations of the algebra.

3.2.1 Syntax

Let $Ev = \{e, f, \ldots\}$ be an alphabet of symbols of *(ordinary) events*, $\overline{Ev} = \{\bar{e}, \bar{f}, \ldots\}$ be symbols of *non-events* and $\Delta_{Ev} = \{\delta_e, \delta_f, \ldots\}$ be symbols of *deadlocked events*. Let us denote $\widehat{Ev} = Ev \cup \overline{Ev} \cup \Delta_{Ev}$. Symbols of $\widehat{Ev}$ are combined into formulas by operations ; (*precedence*), $\bigtriangledown$ (*exclusive or, alternative*), $\|$ (*concurrency*), $\vee$ (*disjunction, union*), $\rrbracket$ (*"not occur"*), $\tilde{\rrbracket}$ (*"mistaken not occur"*). We introduce $Act = \{a, b, \ldots\}$, an alphabet of *action* symbols *(labels)*. A *global labelling function $lab : Ev \rightarrow Act$* binds an action with each event. The function is extended to $\overline{Ev} \cup \Delta_{Ev}$ as follows: $lab(\bar{e}) = \overline{lab(e)}$ and $lab(\delta_e) = \delta_{lab(e)}$.

A *formula* of $AFLP_2$ in a basis $\widehat{Ev}$ is defined by the following production system.

$$E ::= \ e \mid \bar{e} \mid \delta_e \mid \rrbracket E \mid \tilde{\rrbracket} E \mid E; F \mid E\|F \mid E \bigtriangledown F \mid E \vee F$$

Here $e \in Ev$, $\bar{e} \in \overline{Ev}$, $\delta_e \in \Delta_{Ev}$ are *elementary formulas*. We denote by **AFLP$_2$** *a set of all formulas* of $AFLP_2$.

Let E and E' be formulas of $AFLP_2$. E and E' are *isomorphic*, notation $E \simeq E'$, if these formulas coincide up to associativity rules w.r.t. $;, \|, \vee, \bigtriangledown$ and commutativity rules w.r.t. $\|, \vee, \bigtriangledown$.

3.2.2 Semantics

Denotational semantics

A semantics of a process specified by a formula P of $AFLP_2$, notation $\mathcal{D}_{FL2}[P]$, is a set of partial multisets with ordering by dependence relation.

We introduce two equivalences w.r.t denotational semantics. The first is usual one ($=_{\mathcal{D}_{FL2}}$), which means coincidence of pomset sets corresponding to the formulas to be compared. The second is observable one ($=_{\mathcal{D}_{FL2}^+}$), which resembles the previous equivalence with exception it abstracts from non-actions and deadlocked actions.

Axiomatization

In accordance to the equivalence w.r.t. denotational semantics, an axiom system Θ_{FL2} is proposed. It is the same as Θ_{F2} (only the formulas of $AFLP_2$ are used).

A notation $E =_{\Theta_{FL2}} E'$ means that the equation may be proved using the axiom system Θ_{FL2}.

Canonical form of formulas

The definition of canonical form is the same as that for AFP_2.

The following theorems present the required completeness result for Θ_{FL2}.

Theorem 3.2.1 *Every formula of $AFLP_2$ may be proved equal to unique up to isomorphism canonical form using Θ_{FL2}.*

Theorem 3.2.2 *For any formulas E and E' of $AFLP_2$ the following statement is valid: $E =_{\mathcal{D}_{FL2}} E' \Leftrightarrow E =_{\Theta_{FL2}} E'$.*

Hence, we can find whether any two formulas E and E' of $AFLP_2$ equivalent w.r.t. denotational semantics. To do this, it is sufficient to reduce them to their canonical forms F and F' and check them by isomorphism.

For automatization of this process, it is possible to use program with respect of formula labelling.

Operational semantics

Operational semantics of $AFLP_2$, denoted by $\mathcal{O}_{FL2}$, is based on transition systems with rules corresponding to occurrences of pomsets. It is essentially the set of pomsets, after occurrence of which the initial canonical form of formula is reduced to one of the "terminal" formulas s.t. no rule can be applied to them.

Theorem 3.2.3 *Let E be a formula of $AFLP_2$. Then $\mathcal{O}_{FL2}[E] = \mathcal{D}_{FL2}[E]$.*

3.2.3 Equivalences on weakly labelled A-nets

Like we introduced labelling on formulas of AFP_2 and obtained calculus $AFLP_2$, the labelling on AFP_0-formulas may be introduced and a new algebra $AFLP_0$ may be obtained as a result. Since formulas of AFP_0 specify finite A-nets, the formulas of $AFLP_0$ will specify finite *weakly labelled A-nets* (i.e. A-nets having labelling function which may be noninjective).

Let us define a mapping $\Psi_L : \mathbf{AFLP_0} \to \mathbf{AFLP_2}$ as follows.

1. $\Psi_L(e) = e$,

2. $\Psi_L(E;_{FL0} F) = E;_{FL2} F$,

3. $\Psi_L(E\|_{FL0}F) = E\|_{FL2}F$,

4. $\Psi_L(E \bigtriangledown_{FL0} F) = E \bigtriangledown_{FL2} F$.

The symbol "$FL0$" marks the operations of $AFLP_0$, and the symbol "$FL2$" is used for $AFLP_2$ ones. Denotational semantics of $AFLP_0$ is a mapping $\mathcal{D}_{FL0}$, which associates with every formula E of the algebra a set of maximal C-subnets (O-subnets, in terms of [91]) of finite A-net N, specified by the formula. Let us note that with every causal net $C = \langle P_C, T_C, F_C, l_C \rangle$ we can associate lposet $\rho_C = \langle T_C, \prec_C \cap (T_C \times T_C), l_C \rangle$.

Theorem 3.2.4 *Let E be a formula of $AFLP_0$ and F be a formula of $AFLP_2$ s.t. $F = \Psi_L(E)$. Then $\{\rho_C \mid C \in \mathcal{D}_{FL0}[E]\} = \mathcal{D}_{FL2}^+[F]$.*

Hence, with every formula E of $AFLP_0$ which specifies finite weakly labelled A-net N, we can associate the formula F of $AFLP_2$ s.t. the set of lposets of maximal C-subnets of N coincides with the set of lposets of maximal deterministic (sub)processes of the nondeterministic process specified by F. Let us note that the result of the theorem is valid for any (not only maximal) initial C-subnets of N and for any deterministic processes specified by F. In such a case, the initial deterministic processes will correspond to initial C-subnets.

Let us note also that a mapping Ψ_L only replaces operations of $AFLP_0$ by $AFLP_2$ ones. Consequently, if we have finite weakly labelled A-net N specified by $AFLP_0$-formula E, we can analyze its behavior by means of the same $AFLP_2$-formula E.

Now we will consider the equivalences on weakly labelled A-nets. Unlike A-nets, where most of the equivalence notions are merged, interrelation of the equivalences on weakly labelled A-nets is as well as on nets without any restrictions and it may be represented by the same graph.

Theorem 3.2.5 *Let N and N' be weakly labelled A-nets and $\leftrightarrow \in \{\equiv, \underleftrightarrow, \simeq\}$, $\star, \star\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST, prST, pwh, pomh, prh, mes, occ\}$. Then $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow_{\star\star} N'$ iff there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow_{\star\star}$ in the graph in Figure 1.2.*
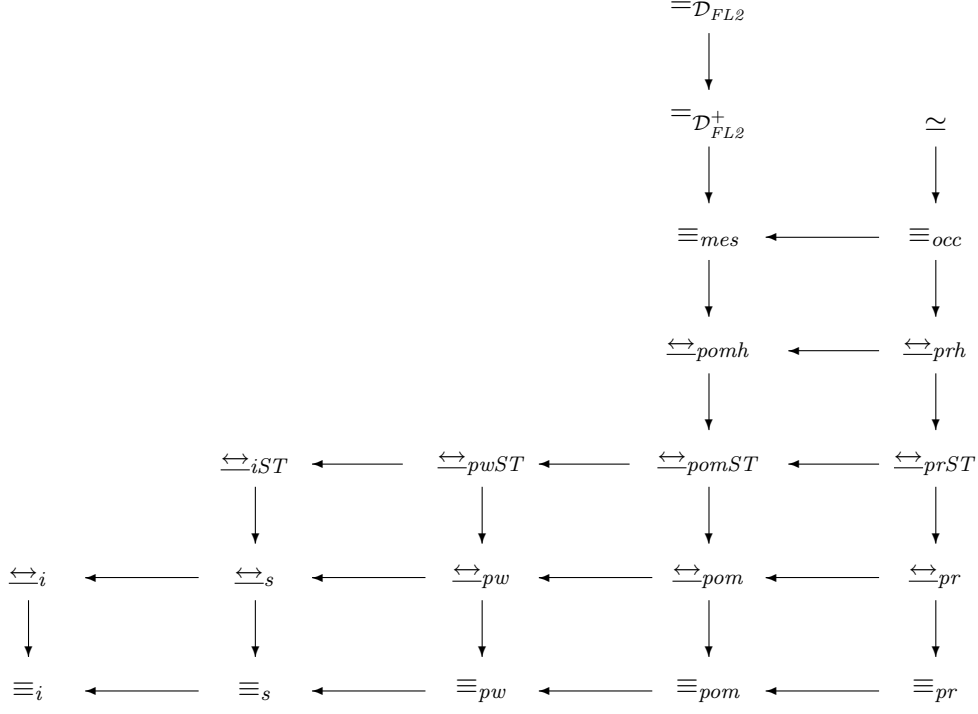
3.2.4 Comparing the basic net and algebraic equivalences

Any finite A-net, as it was proved in [183, 184], can be represented by AFP_0-formula using regularization algorithm. Therefore, any finite weakly labelled A-net can be represented by $AFLP_0$-formula with the use of the analogous algorithm. In the previous section the mapping Ψ_L was defined which associates $AFLP_2$-formula with every $AFLP_0$-formula and preserves the sets of lposets. Hence, one can associate $AFLP_2$-formula E with every finite weakly labelled A-net N s.t. the set of lposets of initial C-subnets of N coincides with the set of lposets of deterministic processes specified by E.

In such a case, it is clear that the concepts of formula equivalences of $AFLP_2$ may be extended to nets. Given some formula equivalence, we will consider two nets to be equivalent iff the formulas are equivalent which are associated with these nets.

Theorem 3.2.6 *Let N and N' be weakly labelled A-nets and $\leftrightarrow \in \{\equiv, \underleftrightarrow, \simeq, =\}$, $\star, \star\star \in \{\_, i, s, pw, pom, pr, iST, pwST, pomST, prST, pwh, pomh, prh, mes, occ, \mathcal{D}_{FL2}, \mathcal{D}_{FL2}^+\}$. Then $N \leftrightarrow_\star N' \Rightarrow N \leftrightarrow_{\star\star} N'$ iff there exists a directed path from $\leftrightarrow_\star$ to $\leftrightarrow_{\star\star}$ in the graph in Figure 3.4.*

$$=_{\mathcal{D}_{FL2}}$$

$$\downarrow$$

$$=_{\mathcal{D}_{FL2}^+} \qquad\qquad \simeq$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\equiv_{mes} \longleftarrow \equiv_{occ}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\underline{\leftrightarrow}_{pomh} \longleftarrow \underline{\leftrightarrow}_{prh}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\underline{\leftrightarrow}_{iST} \longleftarrow \underline{\leftrightarrow}_{pwST} \longleftarrow \underline{\leftrightarrow}_{pomST} \longleftarrow \underline{\leftrightarrow}_{prST}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\underline{\leftrightarrow}_{i} \longleftarrow \underline{\leftrightarrow}_{s} \longleftarrow \underline{\leftrightarrow}_{pw} \longleftarrow \underline{\leftrightarrow}_{pom} \longleftarrow \underline{\leftrightarrow}_{pr}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\equiv_{i} \longleftarrow \equiv_{s} \longleftarrow \equiv_{pw} \longleftarrow \equiv_{pom} \longleftarrow \equiv_{pr}$$

Figure 3.4: Interrelations of the basic net and formula equivalences

3.2.5   Defining of algebraic analogues of the basic net equivalences

In this section, we introduce the basic net equivalences directly on formulas of $AFLP_2$, using transition rules designed to define operational semantics.

Comparing the net equivalences with their algebraic analogues

Theorem 3.2.7 *Let E be a formula of $AFLP_2$ corresponding to finite weakly labelled A-net N, E' be a formula of $AFLP_2$ corresponding to finite weakly labelled A-net N' and $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$, $\star \in \{i, s, pw, pom, iST, pwST, pomST, pwh, pomh, mes\}$. Then $N \leftrightarrow_\star N' \Leftrightarrow E \leftrightarrow_\star E'$.*

The question arises after defining analogues of the net equivalences on $AFLP_2$-formulas, whether some of these equivalences are congruences w.r.t. operations of the algebra. We have found an example of formulas demonstrating that none of the considered equivalences on $AFLP_2$-formulas is a congruence, with the exception of $=_{\mathcal{D}_{FL2}}$, i.e. it is the weakest relation which is a congruence.

Conclusion

Let us mention the **main results** obtained.

1. In the framework of Petri nets without and with silent transitions, a number of behavioral equivalences have been proposed and investigated. The interrelations of the mentioned equivalences have been established on different net subclasses. A logical characterization of a number of equivalence relations has been done. A possibility to use several equivalence notions for effective history preserving reduction of nets has been demonstrated. A compositional approach to comparing behavior of concurrent systems has been treated.

2. For time Petri nets without and with silent transitions, a number of time, untime and region equivalences have been introduced. Their correlation on several subclasses of time nets has been clarified. A regional characterization of the time equivalences has been presented which simplify their check. A stability of the equivalence relations during compositional design of systems has been examined.

3. An extension of the known calculus AFP_2 by labelling function has been proposed: an algebra of labelled nondeterministic concurrent processes $AFLP_2$. An operational characterization as well as complete and sound axiomatization of the semantical equivalences of the mentioned algebras has been provided. A comparison of the algebraic and the net equivalences as well as their transferral from one model to another and back has been done. A method of automatic check of algebraic formulas of the algebras for semantic equivalence has been developed and implemented.

Thus, the basic **conclusions** are the following. A wide set of behavioral equivalence relations have been introduced and systematically investigated for three very important models of concurrency: standard Petri nets, time Petri nets and process algebras. The results obtained provide one with a possibility to evaluate and choose the most suitable formalism and an equality criteria during the process of design of concurrent and distributed systems as well as real time systems.

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